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Developing a local instruction theory for the scale concept: A design research study in Indonesian realistic mathematics education contextualized by Bayt Al-Qur'an Al-Akbar

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Abstract
Proportional reasoning is a foundational concept in middle school mathematics. However, many students struggle because they apply additive thinking rather than multiplicative reasoning when faced with abstract, formula-driven instruction. This study aims to develop and validate a local instruction theory (LIT) for the concept of Scales using the realistic context of Bayt Al-Qur'an Al-Akbar. Employing a design research methodology of the validation study type, this research engaged 39 Grade VII students in Palembang, South Sumatra, through a pilot experiment and a main field test. Data was collected through classroom observations, video recordings, and student worksheets and analyzed using a retrospective analysis method. The findings demonstrate that using the giant, non-proportional dimensions of wooden carving successfully triggered critical cognitive conflicts, prompting students to transition from informal estimates to the formal abstraction of a standardized scale factor. This study suggests that large-scale cultural architecture can serve as a tactical pedagogical intervention to subvert procedural misconceptions, offering a transferable framework for locally culture-based mathematics education globally.

Keywords: Bayt Al-Qur'an Al-Akbar; design research; Indonesian realistic mathematics education; local instruction theory; scale concept

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I. Introduction

Mathematics education is a crucial aspect of schooling, aiming to develop students' logical, analytical, and systematic thinking and problem-solving skills (Lutfi et al., 2024; Setyaningsih & Nurhidayati, 2024; Astuti, 2021). Developing mathematical thinking, including the ability to reason and prove, is central to achieving deeper understanding (Siregar, Suryadi, Prabawanto, & Mujib, 2022; Novikasari & Dede, 2023). Specifically, fostering students' adaptive reasoning, the ability to think logically and justify strategies, is vital, as effective teaching methods can significantly improve this ability (Hartati & Gozali, 2025). In this context, one essential topic at the Junior High School (SMP) level is the Topic of Scales. This material is highly relevant to everyday life contexts (Siregar et al., 2022) and

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equips students to handle quantitative situations encountered in the real world, thus serving as a foundational concept for further scientific and mathematical study (De Lange, 2003; Huzaimah & Risma, 2021).

However, proportional reasoning, which underpins the Scale concept, remains a complex area for Indonesian students, often showing variation in reasoning levels and difficulties in application (Sari, Fiangga, & El Milla, 2023). Specifically, when confronted with scale problems, students frequently fall into a 'proportional trap' in which they mistakenly apply additive thinking rather than multiplicative reasoning. For example, when scaling up a geometric shape or calculating map distances, many students add or subtract a constant value to the dimensions rather than multiplying or dividing by the scale factor. This difficulty arises because traditional classroom practices focus heavily on cross-multiplication formulas without fostering the foundational concept of constant ratios, leaving students unable to apply it when the context is slightly altered.

When students fail to develop their mathematical understanding, learning often degenerates into a series of procedures and imitations of examples without deep meaning (Tesmeri, L., 2025). Despite the relevance of Scales to daily life, many students perceive mathematics as a difficult and abstract subject (Susanti, 2020; Mustakim, 2020). This difficulty is frequently attributed to a lack of relevant resources or teaching materials that help students build conceptual understanding (Sutarni, Sutama, Prayitno, Sutopo, & Laksmiwati, 2024; Sahara, Dolk, Hendriyanto, Kusmayadi, & Fitriana, 2024), as well as monotonous and traditional learning processes that fail to demonstrate connections to the real world. This challenge necessitates continuous innovation in learning trajectories and teaching materials, with a focus on developing instructional designs that effectively contextualize abstract concepts to align with students lived experiences (Febrianto & Sari, 2024).

To address these challenges, there is a need to create meaningful learning experiences by promoting students' active involvement in constructing their own understanding (Febrianto & Sari, 2024). The Indonesian Realistic Mathematics Education (PMRI) approach, adapted from the RME theory initiated by Hans Freudenthal, offers a relevant and proven framework (Anderson, Tupamahu, Sabandar, Makaruku, & Sabandar, 2024). Prior studies have demonstrated that PMRI is effective in enhancing students' mathematical thinking (Mulbasari, Putri, Zulkardi, & Aisyah, 2024; Herlawan, Armin, & Poa, 2023; Halimah, Riyadi, & Atmojo, 2022; Ndiung, 2021). The philosophical core of PMRI is often illustrated by the Iceberg Model, which asserts that the visible, formal mathematics (the tip of the iceberg, or Formal Knowledge) must be supported by the invisible process of informal strategies, modelling (Model-of and Model-for), and conceptual reasoning (the submerged part of the iceberg) (Sari, Fiangga, El Milla, Shahrill, & Yanti, 2025).

Prior studies have extensively utilized various realistic contexts to teach scales and ratios. For instance, researchers have employed map navigation, photo resizing, and miniature models to bridge the gap between informal and formal scaling concepts (Siregar et al., 2022; Huzaimah & Risma, 2021). While these studies successfully improved student procedural skills, they frequently bypassed the cognitive conflict needed to realize why a standardized scale factor is mathematically necessary, often presenting pre-scaled objects directly to students. Furthermore, while ethnomathematics in South Sumatra has explored local textiles such as Songket and traditional houses (Kurnia, Agustiani, & Ramury, 2023; Rohmah, Rohim, & Asmana, 2024), research on giant religious-cultural landmarks remains nonexistent.

This study addresses these gaps by implementing the unique context of Bayt Al-Qur'an Al-Akbar (the world's largest wooden Qur'an carving located in Palembang, South

Sumatra) as the realistic context for Scales (see Figure 1).

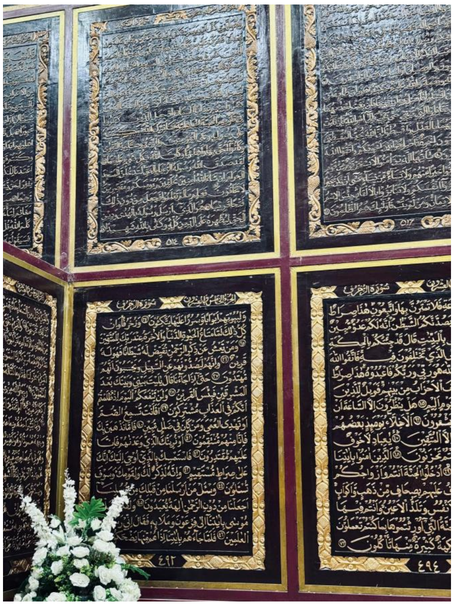

Figure 1. The architectural context of Bayt Al-Qur'an Al-Akbar Palembang

Local contexts rich in patterns and proportions, such as cultural architecture, are ideal for understanding this topic (Supriadi, Astutik, & Pamungkas, 2025), thereby making learning more meaningful (Serepinah & Nurhasanah, 2023; Siregar et al., 2024).

The strategic novelty of this research lies in utilizing the inherent non-proportionality between a student's personal Qur'an and the giant carved pages. Unlike previous research that uses perfectly proportional models, this design deliberately induces a cognitive conflict when students discover that the length-to-width ratios do not match. This forces them to naturally abstract the necessity of a single, standardized ratio (Scale). Consequently, this research makes a significant theoretical contribution by expanding the PMRI literature through a newly validated Local Instruction Theory (LIT) derived from a large-scale cultural artefact.

Therefore, this study aims to develop and validate a PMRI-based LD on Scales material using the Bayt Al-Qur'an Al-Akbar context through a design research validation study. The results of this research are expected to provide practical contributions to teachers by offering a tested instructional model and theoretically enrich the study of local culture-based mathematics learning by providing a validated LIT for the Scales topic.

II. Research Method

This study adopted the Design Research (DR) methodology, specifically the Validation Study type (Prediger, 2024). This approach was chosen because it allows for the systematic development, iterative testing, and refinement of the instructional design LD while contributing to the formation of a LIT regarding educational interventions (Prahmana, Sagita, Hidayat, & Utami, 2020). The validation study is proven effective for developing and validating theories of educational intervention and learning trajectories that enhance educational quality (Gravemeijer & Cobb, 2006).

The research procedure followed the three main, iterative stages of Design Research: (1) Preliminary Design, (2) Design Experiment, and (3) Retrospective Analysis, aiming to bridge theoretical ideas with authentic classroom practices (Prediger, 2024; Plomp & Nieveen, 2013). Data collection during the Design Experiment phase utilized classroom observations, including field notes and video recordings (documentation), to capture LD implementation, student activities, teacher-student interactions, and alignment with the HLT (Firsta, Meryansumayeka, Susanti, & Zulkardi, 2024).

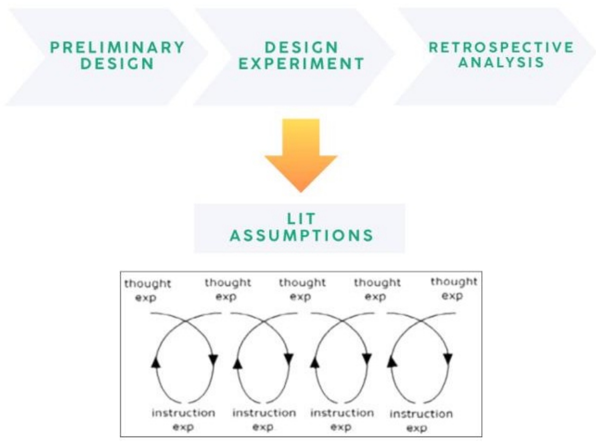

Figure 2. The design research cycle

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The research subjects were Grade VII students (approximately 12–13 years old) in the 2025/2026 academic year at an undisclosed junior high school in Palembang, South Sumatra. A total of 39 students participated in the research, comprising 8 in the pilot experiment phase and 31 in the main teaching experiment (Field Test). The subject selection was purposive to ensure representation of high, medium, and low mathematical abilities, which is essential for capturing the full spectrum of students' emergent thinking strategies and informal modelling behaviours. Deeply embedded in local Palembang culture, these students were highly familiar with the Bayt Al-Qur'an Al-Akbar as a cultural and religious landmark, having either visited it or seen it in local media. This baseline cultural familiarity was critical; it eliminated the cognitive load of understanding an unfamiliar context, allowing students to focus on the mathematical attributes of the artefact immediately. However, despite their cultural proximity, none of the students had previously analyzed the landmark from a proportional or scaling perspective, making it a fertile ground for progressive mathematization.

Data analysis was conducted using a retrospective approach, in which the collected qualitative data were systematically compared with the generated Hypothetical Learning Trajectory (HLT). The video recordings of classroom interactions and field notes were transcribed and coded to identify critical moments of learning, shifts in mathematical strategy, and instances of cognitive conflict. Students' written artefacts on the worksheets (LKM) were analyzed using a constant comparative approach to evaluate their progression through the PMRI levels: from situational (informal estimation), to experiential modelling (model-of and model-for), and finally to formal mathematical reasoning. Triangulation among student work, video data, and researcher field notes ensured the validity and reliability of the retrospective analysis used to refine the final LIT.

III. Results and Discussion

Preliminary Design and Validation

This stage aims to design a series of student-centred learning activities that could improve mathematical reasoning (Rawani, Putri, Zulkardi, & Susanti, 2023). Key activities include a literature review on PMRI, an analysis of the concepts of Scale and Ratio, and an analysis of the potential context of Bayt Al-Qur'an Al-Akbar for initiating mathematization. The core output of this stage is the development of LD and HLT.

After the student worksheets (LKM) were developed, expert validation was conducted through a Focus Group Discussion (FGD) involving lecturers interested in PMRI and practising seventh-grade mathematics teachers. The FGD focused on validating the LKM using content validity, construct clarity, and linguistic appropriateness.

The FGD resulted in crucial revisions to the LKM before its implementation. Figure 3 highlights the revisions made to the LKM's content focus and linguistic terminology, while Figure 4 illustrates the key conceptual changes to the contextual measurement activity.

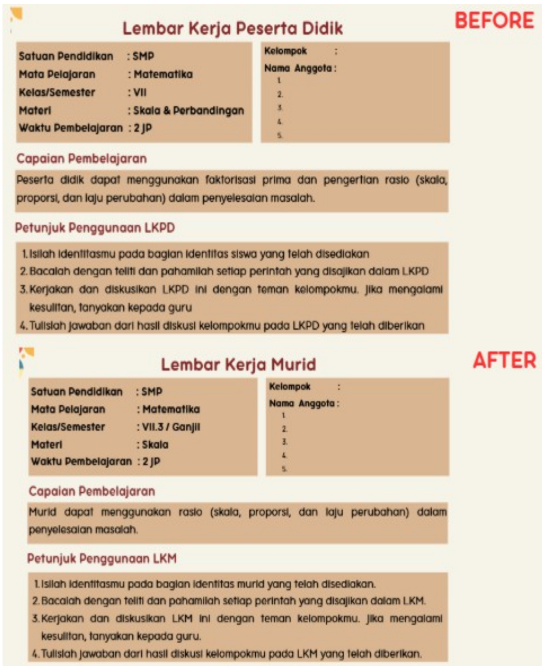
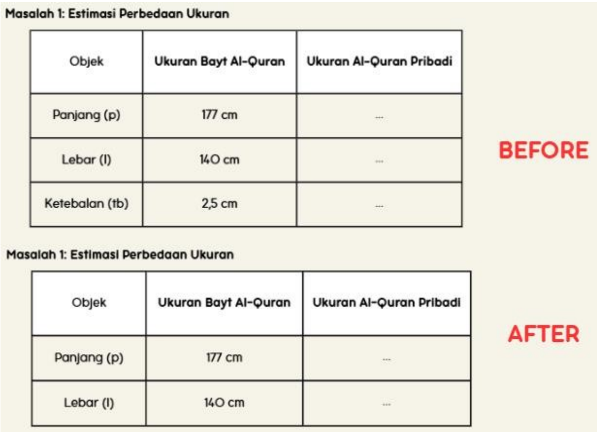


Figure 3. Comparative view of LKM revisions: pedagogical and content focus

Revisions about pedagogical and content focus ensured better curriculum alignment and instructional clarity. The learning objective was refined to exclusively emphasize the application of ratios and proportions, moving away from a previous focus on prime factorization, and the instructional focus was simultaneously tightened to ensure coherence with the concepts of Scales. Furthermore, to maintain linguistic consistency, the administrative term 'peserta didik' was consistently replaced with the more current pedagogical term 'murid' throughout the document. Finally, the activity prompt was modified to explicitly request the student's Grade VII class and semester for accurate subject tracking during the experiment phase.


Masalah 1: Estimasi Perbedaan Ukuran

Objek	Ukuran Bayt Al-Quran	Ukuran Al-Quran Pribadi
Panjang (p)	177 cm	...
Lebar (l)	140 cm	...
Ketebalan (tb)	2,5 cm	...

Masalah 1: Estimasi Perbedaan Ukuran

Objek	Ukuran Bayt Al-Quran	Ukuran Al-Quran Pribadi
Panjang (p)	177 cm	...
Lebar (l)	140 cm	...

Figure 4. Comparative view of LKM revisions: contextual measurement activity

Conceptual revisions were implemented to strengthen the design's focus on proportional reasoning. To prevent computational distraction from non-integer (decimal) data (a core PMRI principle), the original activity that required measuring the thickness of Qur'an pages was removed. The activity was subsequently simplified to focus solely on estimating the size difference using the length and width of the personal and giant Qur'ans.

These strategic revisions confirmed the overall design's theoretical soundness before implementation (Plomp & Nieveen, 2013). Following the validation and subsequent revisions, the finalized LD and its guiding tool, the HLT, were ready for the Design Experiment phase. The HLT details the conjectured learning process, providing the sequence of activities and anticipated student strategies (Conjectures) for the Scales material, contextualized by Bayt Al-Qur'an Al-Akbar.

Table 1. Hypothetical learning trajectory (HLT) for the scale concept using Bayt Al-Qur'an Al-Akbar

Aim	Activity	Conjectures
Students will be able to read and identify quantitative information from a text.	Introducing the Bayt Al-Qur'an Al-Akbar Palembang context (Questions 1 & 2).	Students will record the specified dimension data from the text and answer questions about the cultural context (Situational Phase).
Students will be able to perform measurements on real objects.	Size Difference Estimation (Activity 1): Students measure the length and width of their personal Qur'an (see Figure 3).	Students will obtain actual-size data (e.g., $20\ cm \times 15\ cm$) and notice the massive size difference compared to the giant artefact.
Students will be able to determine the comparison (ratio) between two similar quantities.	Activity 1: Determining Ratio and Scale (Table Columns): Filling in the size data and finding length-to-length and width-to-width ratios.	Students will find the ratio of lengths (Giant L: Personal L) and widths (Giant W: Personal W) and attempt to simplify them into 1: x format.
Students will be able to compare the similarities and differences between ratios.	Activity 1: Determining Ratio and Scale (Questions below the table): Comparing the calculated length and width ratios.	Students discover that $Ratio Length \neq Ratio Width$, concluding the comparison is non-proportional. This triggers a cognitive conflict.
Students will be able to define the concept of scale from the context.	Activity 2: Defining Scale (Question 1): Formulating the definition of scale in their own words based on the conflict resolution.	Students formulate the definition of scale as a standardized ratio between the model/map and the actual size (Formalization Phase).

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Students will be able to use the scale formula to calculate actual distance.	Activity 2: Application Task (Question 2): Calculating the actual distance on a Palembang City map with a scale of 1: 20,000.	Students apply the formula: $\text{Actual distance} = \frac{\text{Dist on map}}{\text{Scale}}$, and carefully convert units from cm to km (Formal Phase).

Figure 4. Student worksheet (LKM) for the introductory activity

Figure 5. Student worksheet (LKM) showcasing the elicitation of informal size ratio estimation

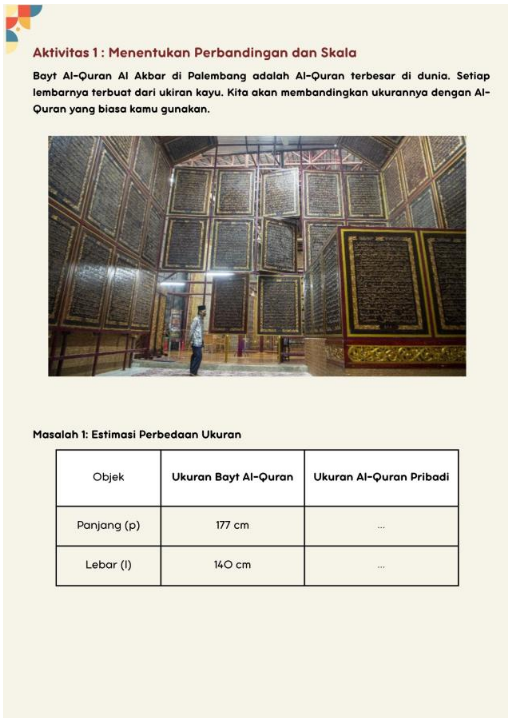

Figure 6. Student worksheet (LKM) detailing the physical measurement and ratio tracking table

Figure 7. Student worksheet (LKM) demonstrating the student's text explanation regarding non-proportionality

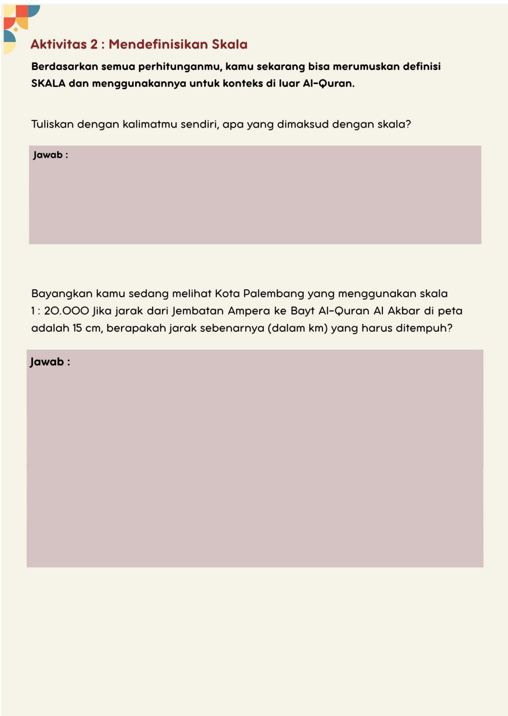

Figure 8. Student worksheet (LKM) showing the self-formulated definition of scale and the application task

Design Experiment

This stage was carried out in two cycles: the Pilot Experiment (First Cycle) and the Teaching Experiment (Second Cycle/Field Test). This phase served to implement the developed LD and validate the HLT regarding the concept of scale, using the Bayt Al-Qur'an Al-Akbar Palembang as a rich context.

1st Cycle

Activity: Introducing Bayt Al-Qur'an Al-Akbar Palembang (Questions 1 & 2)

The objective of the introductory activity was to enable students to read and extract key quantitative and qualitative information from the provided text about the Bayt Al-Qur'an Al-Akbar. This activity guided students into the situational phase of the HLT, ensuring they recognized the cultural context before engaging in mathematical abstraction.

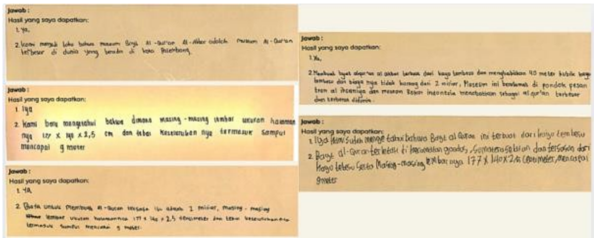

Figure 9. Student's initial extraction of quantitative information from the contextual text

The analysis of student responses confirmed that the contextual text successfully guided them into the situational phase. Most groups were able to extract complex quantitative information, such as the page dimensions ($177\ cm \times 140\ cm \times 2.5\ cm$) and the overall thickness ($9\ m$). This immediate exposure to large-scale, real-world numbers set the stage for the next activity involving size comparison and the concept of scale.

Activity: Introduction Size Difference Estimation / Informal Reasoning (Question 3)

The goal of this activity was to elicit students' informal proportional reasoning by asking them to estimate the ratio of the giant Qur'an's size to their personal Qur'an's size. This served as a bridge from the situational context to the informal modelling phase.

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Coba bandingkan dengan Al-Quran yang kamu bawa. Kira-kira, berapa kali lipat lebih besar lembaran Al-Quran raksasa ini dari lembaran Al-Quran milikmu? Jelaskan mengapa kamu memilih angka itu ?

Jawab:
: Al-Qur'an terbesar 8 kali lipat lebih besar dari Al-Qur'an yg kami bawa. karena Al-Qur'an terbesar panjangnya 177 cm dan lebarnya 140 cm Sedangkan al-Qur'an kami memiliki panjang 21 cm dan lebar-nya 15 cm.

Figure 10. Student informal reasoning strategies showing the spontaneous division method and size ratio estimation

The analysis of student responses in the estimation activity validated the HLT's conjecture regarding the initiation of informal proportional reasoning. Two key informal strategies were observed among the groups. First, students displayed a Focus on Length as the primary dimension for comparison, demonstrating a natural tendency to simplify the complex two-dimensional ratio into a one-dimensional problem. This led to a spontaneous division strategy in which students performed an early, informal ratio calculation by comparing the length of the giant Qur'an (177 cm) with their personal Qur'an (e.g., $177 \div 21 \approx 8.4$). Second, an Estimation Bias was evident, characterized by the dominance of the "8 times larger" approximation, despite slight variations in the sizes of their personal Qur'ans. This suggests that students prioritised a round-number approximation based on the magnitude difference over precise calculation, thereby verbally expressing an initial, informal ratio. This successful activation of informal modelling provides the necessary foundation for students to transition to the formal determination of a precise single ratio (Scale) in the subsequent activity.

Activity 1: Determining Ratio and Scale (Formal Modelling)

The objective of Activity 1 was to guide students from informal estimation toward formal mathematical modelling by having them measure their personal objects and calculate the respective ratios. This stage aimed to challenge their initial assumptions about proportionality and formally introduce the concept of scale as a necessary standardized ratio.

Analysis of Measurement and Calculation. In this activity, students were required to complete the table by measuring the dimensions of their personal Qur'an.

Masalah 1: Estimasi Perbedaan Ukuran

Objek	Ukuran Bayt Al-Quran	Ukuran Al-Quran Pribadi
Panjang (p)	177 cm	21 cm
Lebar (l)	140 cm	15 cm

Figure 11. Student work detailing the measurement

All groups consistently reported the size of their personal Qur'an as 21 cm (length) and 15 cm (width) in the table. This consistency, likely influenced by group agreement or minor teacher scaffolding (as noted in the Method section on standardization), yielded a uniform dataset for analysis. All student groups successfully calculated the length-to-length and width-to-width ratios and correctly simplified them to the 1:x format as requested.

To illustrate how students navigated this measurement and encountered the core mathematical conflict, the following dialogue fragment from Group 3 during Activity 1 is presented:

Student A: The length of our personal Qur'an is 21 cm, and the width is 15 cm. Now we must compare it to the giant one (177 cm × 140 cm).
Student B: Okay, let's divide them. For the length, $177 \div 21$ is about 8.4. If we simplify it into a 1:x format, it's roughly 1:8.4
Student C: Wait, now look at the width. $140 \div 15$ is 9.3. So, the ratio is 1:9.3. They are not the same!
Student B: Ah, you're right. 1:8.4 is not equal to 1:9.3. If they are different, it means our personal Qur'an is not a perfectly shrunk version of the giant one. It's slightly distorted.
Student A: Exactly! That is why we can't just pick any random numbers to make a map or a model. We need a single fixed ratio. That must be what a 'Scale' is for.

This dialogue confirms that the structured measurement task successfully sparked an internal debate, leading students to dynamically recognize non-proportionality and verbalize the transition from informal data tracking to the need for a formal scale.

The Crucial Finding: Non-Proportionality. The most critical finding validates the core conjecture of the HLT: the activity successfully challenged students' prior informal assumptions by revealing a difference between the two ratios.

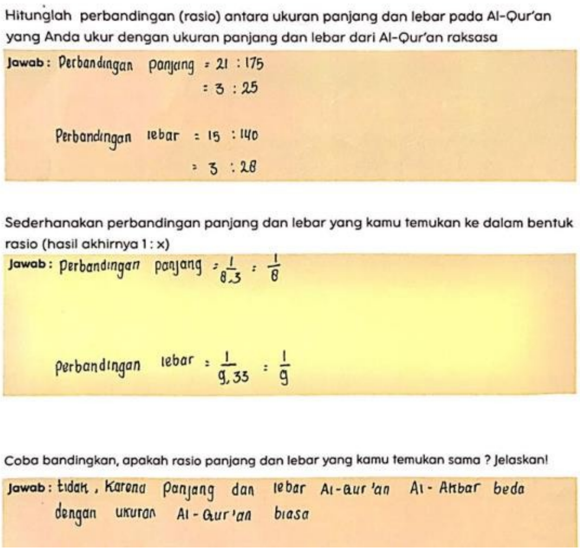

Figure 12. Student response demonstrating the recognition of non-proportionality between the two calculated ratios

All groups unanimously answered, "Tidak" (No). All explanations correctly stated that the two ratios (Length Ratio and Width Ratio) were different.

This finding confirms that the contextual activity successfully created a cognitive conflict necessary for the next step of mathematization. Students were confronted with two different values, proving that their personal Qur'an was not a perfectly proportional scaled model of the Bayt Al-Qur'an Al-Akbar.

This step is vital for formulating LIT. The realization that:

$$Ratio_{Length} \neq Ratio_{Width}$$

Forced students to confront the need for a single, standardized comparison factor, which cannot be achieved using random comparison. This directly validates the HLT's conjecture that comparing non-proportional objects leads students to conclude that a specific, standardized ratio is required, thus driving them toward the formal definition of scale. The successful execution of this activity indicates that students were effectively transitioned into the Formal Modelling Phase, setting the stage for the abstraction and definition of scale in the subsequent activity.

Activity 2: Defining Scale and Application

The final sequence of the Learning Design aimed to facilitate the transition from formal modelling to Formalization and Abstraction of the Scale concept, followed by its application in a new context.

The analysis of student responses confirmed the success of the trajectory, as all groups were able to formulate a conceptually sound definition of scale using their own language, directly addressing the limitations observed in Activity 1. Furthermore, students were able to state the corresponding mathematical formula explicitly.

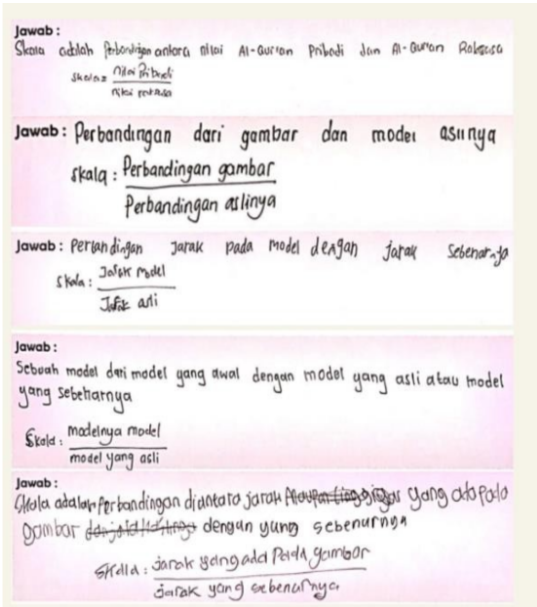

Figure 13. Student's definition of scale and the corresponding formula

This collective achievement validates the HLT's conjecture that confronting the non-proportionality inherent in the previous

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comparison (Activity 1) drive students to recognize the necessity for a standardized ratio (scale). The definitions demonstrate that students have successfully abstracted the concept: Scale is understood as the ratio of a model/image distance to the actual object/distance, confirming the attainment of the Formalization Phase.

A particularly powerful example is the definition provided in activity 2, which, despite its contextual specificity ("Alquran pribadi dalam Alquran raksasa"), explicitly links the formal concept of scale back to the initial contextual experience.

Formal Application and Unit Conversion

The final task in Activity 2 required students to apply the defined scale concept to solve a problem: "Imagine you are looking at the city of Palembang using a scale of 1: 20.000. If the distance from Ampera Bridge to Bayt Al-Quran Al Akbar on the map is 15 cm, what is the actual distance to be travelled (in km)?"

The Design Experiment

During the small group implementation, the task did not explicitly specify the final unit "(in km)". This observation is crucial, as several students successfully calculated the actual distance in centimeters (300.000 cm) but failed to convert the result into kilometers until reminded by the teacher (researcher). This oversight was not due to a lack of computational ability, but rather a temporary breakdown in horizontal mathematization; the student focused intensely on the numerical calculation required by the scale formula but momentarily detached from the realistic context (travel distance is typically measured in kilometers, not centimeters). This finding necessitated a minor but significant revision to the HLT and the LKM.

2nd Cycle

The Retrospective Analysis

Based on the pilot observation, the Activity Sheet was immediately revised before the teaching experiment (Field Test). The final question in Activity 2 was improved by explicitly stating the required unit of measurement (km).

This revision proved effective: during the field test, students completed the scale application task, demonstrating not only the ability to use the scale formula but also the required unit conversion, thereby validating the overall HLT and the refinement process inherent in the design research cycle.

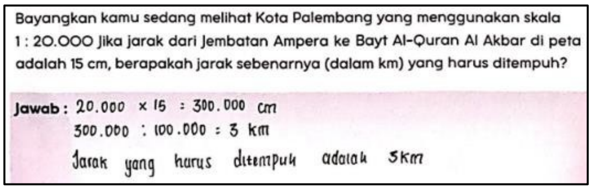

Figure 14. Student's successful application of the scale concept, demonstrating the correct unit conversion from cm to km (post-revision)

All groups provided the correct final answer: 3 km. No groups wrote down the formal mathematical formula; instead, they presented the final answer directly.

This consistent result strongly indicates that students had achieved the desired level of Formal Application. Despite the absence of explicitly written formulas, the accuracy of the final answer confirms they executed the necessary multi-step mathematical procedure:

$$Actual\ Distance = Map\ Distance \times Denominator\ of\ Scale$$
$$= 15\ cm \times 20.000 = 300.000\ cm$$
$$300.000\ cm \div 100.000 = 3\ km$$

Success in this complex problem, involving the multiplication of ratios and subsequent unit conversion (cm to km), without teacher scaffolding provides definitive evidence of the LD's effectiveness in improving mathematical reasoning on the topic. The ability to perform these computations internally and present a correct result demonstrates that the conceptual understanding acquired through the PMRI/RME process has been solidified into an operational mathematical tool.

Retrospective Analysis and LIT

Following the Design Experiment (Field Test), the retrospective analysis focused on validating the developed LIT by comparing the HLT with the Actual Learning Trajectory (ALT) observed in the classroom. The analysis strongly confirmed that the designed sequence of activities successfully fostered students' reasoning abilities, culminating in the construction of the formal Scale concept.

The learning activities, contextualized by the Bayt Al-Qur'an Al-Akbar, successfully triggered both horizontal and vertical mathematization. This evolutionary process in students' thinking is summarized in the Iceberg Model below, which illustrates the connection between students' informal strategies and the formal mathematical concept.

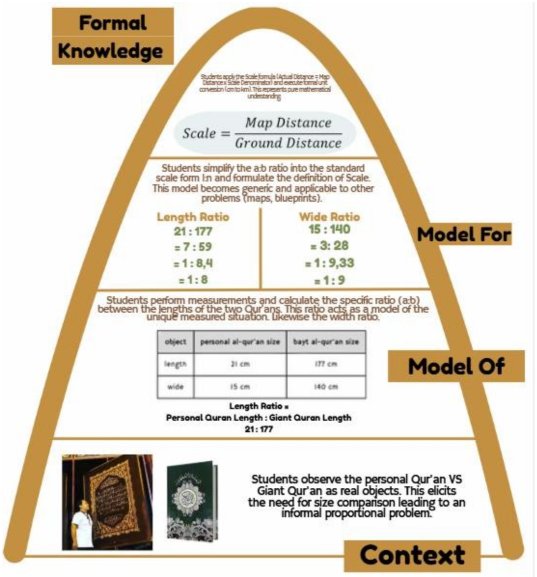

Figure 15. The iceberg model summarizing the validated LIT

The sequence of learning activities designed within the LIT successfully prompted students to construct key ideas about the Scale concept, as revealed by the teaching experiment and retrospective analysis. The research design played a crucial role by providing meaningful context and carefully guiding students to first recognize the non-proportionality inherent in their initial comparisons, which was essential before they could define the necessity of a standardized ratio (Scale). These scaffolding enabled students to connect the principle of proportionality to the real-world size comparison. Once students understood the constraint that random comparison factors yield inconsistent results, they were naturally driven to reason mathematically and formulate the general rule for scale based on their own conceptual needs.

The robust results suggest that the developed LIT had a significant impact on student learning by motivating them to actively discover mathematical principles collaboratively (Tarigan & Rakhmawati, 2024; Herlawan et al., 2023; Siregar et al., 2022; Halimah et al., 2022; Ndiung, 2021). The integration of the Bayt Al-Qur'an Al-Akbar as a tangible cultural artefact provided a richly engaging environment for validating the learning trajectory. By observing, comparing, and analyzing the real-world dimensions, students were able to investigate size relationships concretely and creatively, successfully bridging their informal experiences with formal mathematical abstraction. This aligns perfectly with the core tenets of Realistic Mathematics Education (RME/PMRI), confirming the context as a validated tool for fostering deep conceptual understanding of the Scale concept.

These findings align with and expand upon previous PMRI studies utilizing realistic modelling (Tarigan & Rakhmawati, 2024; Herlawan et al., 2023). Consistent with the trajectory noted by Siregar et al. (2022), this study confirms that starting with a highly tangible, real-world object allows students to initially ground their mathematical arguments in situational reality rather than in abstract symbols. However, while previous interventions by Huzaimah & Risma (2021) directly provided students with pre-proportional maps in which the scale was already defined, our trajectory took a divergent, more critical path. By forcing students to compare the non-proportional dimensions of their personal Qur'an and the giant artefact, this study successfully triggered an explicit cognitive conflict ($ratio_{length} \neq ratio_{width}$).

This crucial tactical difference allowed our students to experience what Gravemeijer & Terwel (2010) describe as 'progressive mathematization' more intensely. Instead of merely consuming a given scale formula, students had to invent the need for a standardized ratio to resolve the structural distortion. From this comparison, we can infer that authentic mathematical reasoning on scales is more robustly developed when students experience a breakdown of random ratios rather than immediate harmony. For future instructional practices, it is highly suggested that mathematics educators do not shy away from using non-proportional real-world pairs; instead, they should actively leverage them as a pedagogical catalyst to guide students away from additive traps and toward true formal multiplicative abstraction.

IV. Conclusion

The retrospective analysis confirms that the developed LIT, guided by the principles of Indonesian Realistic Mathematics Education (PMRI/RME), proved highly effective, validating the entire learning trajectory. The context of the Bayt Al-Qur'an Al-Akbar successfully served as an engaging and validated tool, providing the essential structure for students' conceptual progression. The trajectory effectively scaffolded students from the Situational Phase (verbal estimation of size difference, Model of the comparison) toward formal mathematical abstraction (Model for the concept, resulting in the self-formulated definition of scale and its subsequent Formal Knowledge application). Crucially, the structured activity that resulted in the non-proportional comparison of the length ratio and the width ratio served as a powerful intervention, naturally compelling students to discover the necessity of a standardized ratio (Scale), thus eliminating the need for rote memorization of the formula. The consistent success in both conceptual formulation and complex calculation (achieving the correct 3 km result) provide strong evidence of the effectiveness of the developed Learning Design in improving students' mathematical reasoning on the topic of Scales. Therefore, this research offers a validated LIT using the Bayt Al-Qur'an Al-Akbar context as a potent alternative instructional model for teaching proportional reasoning at the Junior High School level. The validation of this specific LIT, grounded in a novel local context, addresses a critical gap in the contextual PMRI literature.

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