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Difficulties faced by junior high school students in solving integer multiplication problems: A Newman's error analysis approach

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Abstract

Integer multiplication is a foundational arithmetic competency, yet junior high school students persistently struggle to solve word problems involving it, and these difficulties often go unnoticed when only the correctness of final answers is assessed; consequently, it remains unclear at which cognitive stage students' reasoning actually breaks down. To address this specific problem, this study aims to analyze the types of errors eighth-grade students make when solving integer multiplication problems at a public junior high school in Lembata Regency, East Nusa Tenggara. A qualitative descriptive approach was employed, involving eleven students as participants. The instruments used included a written test consisting of ten questions and an interview guide based on the five stages of Newman's Error Analysis (NEA): reading, comprehension, transformation, process skills, and encoding. The analysis revealed no errors at the reading stage. However, errors were identified at the comprehension stage (10.51%), transformation stage (26.46%), process skills stage (28.79%), and encoding stage (34.24%). The highest error rate occurred during the encoding stage, indicating that although students were able to process the problems correctly, they struggled to express the final answers accurately. These findings suggest that students face challenges related to conceptual understanding, procedural fluency, and mathematical expression. The study highlights the need for instructional strategies that reinforce mastery of foundational skills and enhance students' ability to articulate mathematical ideas precisely.

Keywords: difficulties; encoding; integer multiplication; Newman's error analysis; process skills

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I. Introduction

Education plays a vital role in fostering respect for oneself and others, forming the foundation of human dignity. Education that integrates the concept of human dignity not only imparts knowledge but also enables individuals to explore their identities, internalize core values, and develop personal responsibility (Eddahab-Burke and Okur, 2025). Mathematics education plays a crucial role in developing students' logical reasoning and problem-solving abilities. These skills are essential for understanding and navigating real-life situations. Therefore, mathematics instruction should ideally aim to foster deep conceptual understanding, particularly of foundational topics that serve as a basis for mastering more advanced content (Bahbah & Erradi, 2024).

One of the essential competencies in mathematics that every individual must acquire is problem-solving ability. Thus, teachers need to adopt contextual and adaptive instructional approaches that accommodate students' diverse characteristics (Reinke, 2019). However, classroom realities indicate that many students continue to rely heavily on rote memorization when working on mathematical problems, especially those involving integer multiplication. While higher-achieving students may see multiplication as repeated addition, many of their peer's struggle to develop a meaningful conceptual understanding of it (Kurniati, Prabawanto, & Haeruddin, 2022).

International evidence makes clear that difficulty with integer multiplication is a worldwide phenomenon, not a uniquely Indonesian one. In the 2022 Programme for International Student Assessment (PISA), as reported by the OECD (2023), about 31% of students across OECD countries scored below Level 2 in basic mathematical proficiency. In several countries, more than half of 15-year-olds could not, at or below Level 2, correctly handle problems built on basic arithmetic addition, subtraction, division, and integer multiplication. Indonesia sits among the countries with a comparatively large share of students below this baseline, particularly on tasks involving the multiplication of integers and other rational numbers.

The topic of integer multiplication is taught within the Kurikulum Merdeka (Independent Curriculum) framework to address students' difficulties in understanding fundamental mathematics, which is the basis for arithmetic skills (Hidayah, Sa'dijah, Anwar, Yerizon, & Arnawa, 2024). Although a critical topic, integer multiplication is widely regarded as one of the more challenging areas of mathematics. One way to mitigate these difficulties is to embed multiplication problems in real-life contexts to make learning more engaging and relevant (González, 2017). This approach helps students relate mathematical operations to real-world scenarios, thereby enhancing their conceptual understanding.

At the heart of the difficulty with integers is the struggle to develop a genuine conceptual understanding, which often manifests as errors when students multiply them. A common instance is mixing up the signs of positive and negative numbers (Alfarisi, Dasari, Aljupri, & Sikma, 2022; Rosyidah, Maulyda, Jiwandono, Oktaviyanti, & Gunawan, 2021). Mistakes of this kind signal that students have not internalized the underlying structure and principles of integer multiplication and tend to lean on superficial, non-conceptual shortcuts. This makes it important to examine students' cognitive processes more closely when they solve integer-multiplication problems, attending not just to the final answer but to the reasoning steps that lead there.

When students work on mathematics problems, especially word problems built around integer multiplication, the mistakes they produce can be traced to particular points in their thinking process. Newman's Error Analysis (NEA) groups these into five types (Kusmayadi, Sahara, & Fitriana, 2022; Makgakga, 2023; Priliawati, Slamet, & Sujadi, 2019): reading, comprehension, transformation, process skills, and encoding. A reading-stage mistake may appear without necessarily propagating to later stages. A comprehension mistake reflects a student's failure to extract the key information conveyed by the text or accompanying figure. A transformation mistake surfaces when the student cannot turn that information into a suitable mathematical representation. A process-skills mistake involves faulty execution of the chosen procedure or formula, whereas an encoding mistake appears when the student cannot state or finalize the answer correctly.

NEA is widely regarded as a well-suited approach for diagnosing students' mathematical errors because it lays out the cognitive steps of problem-solving in sequence, running from reading through to encoding (Triliana & Asih, 2019). Whereas the structure of the Observed Learning Outcome (SOLO) framework foregrounds the complexity of student responses, NEA offers a stage-by-stage lens that pinpoints exactly where a student falters while solving the problem (Stålne, Kjellström, & Utriainen, 2016). The framework fits integer-multiplication word problems particularly well, since arriving at a solution demands an ordered chain of cognition: decoding the question, interpreting what integer multiplication means in the given context, recasting the verbal information as a mathematical expression, carrying out the computation, and finally recording the answer (Shinariko, Saputri, Hartono, & Araiku, 2020). In this way, NEA does double duty: it pinpoints the kinds of errors students commit and, at the same time, provides a footing for designing instruction that addresses them directly.

Students' mathematical mistakes often emerge once a familiar concept is presented in an unfamiliar format or setting. Kusumadewi & Retnawati (2020) reported that most students are still not used to tackling tasks that call for higher-order thinking; they tend to have trouble identifying key terms, expressing the problem in mathematical language, choosing a workable strategy, and avoiding calculation slips that lead to wrong answers. Such findings imply that the difficulty runs deeper than conceptual understanding alone and also involves limits in logical, systematic reasoning. A diagnostic approach is therefore needed, one that not only labels error types but also traces the particular cognitive stage at which each one arises.

Taken together, the existing body of research establishes a fairly consistent state of the art. Studies applying Newman's Error Analysis (NEA) have repeatedly demonstrated its diagnostic value across a wide range of mathematical domains, including fractions and higher-order thinking tasks (Abdullah, Abidin, & Ali, 2015), mathematical induction and proof (Ahmadi, Kusumah, & Jupri, 2019; Suradi & Djam'an, 2021), quadratic equations (Makgakga, 2023), word problems on linear programming (Lubis & Jupri, 2023), and mathematical literacy at the primary level (Mubarokah & Amir, 2024). Internationally, NEA has a long and well-established research base: cross-national studies have shown that the majority of students' errors on written mathematical tasks occur before the process-skills stage, that is, at the comprehension and transformation levels (Clements & Ellerton, 1992), and later work has confirmed NEA's value as a diagnostic tool linking numeracy and literacy across large student samples (White, 2010). This international evidence establishes NEA as a robust, widely validated framework rather than a locally confined method. In parallel, a separate strand of literature has documented that integer operations remain a persistent source of misconceptions, with errors in handling positive and negative signs and an over-reliance on whole-number reasoning reported among both students and preservice teachers (Alfarisi et al., 2022; Harun, Cuevas, Asakil, Alviar, & Solon, 2023; Rosyidah et al., 2021). However, these two lines of inquiry have largely developed in isolation: NEA studies have tended to focus on algebra, geometry, statistics, and proof, while research on integer multiplication has concentrated on classifying misconceptions without systematically tracing the cognitive stage at which each error originates. This reveals a clear research gap. There is limited evidence on how the five sequential NEA stages manifest specifically in contextual integer multiplication word problems among junior high school students, particularly within the Indonesian *Kurikulum Merdeka* context and in under-researched regions such as Lembata Regency, East Nusa Tenggara. The novelty of the present study therefore lies in bridging these two strands: it applies the full sequential NEA framework directly to integer multiplication word problems, pairing written test data with in-depth interviews to locate precisely where students' reasoning breaks down, from comprehension through to encoding, rather than merely reporting the prevalence of misconceptions. In doing so, the study offers a stage-specific diagnostic profile that can inform targeted, rather than generic, instructional remediation. To state the novelty explicitly, three features distinguish this study from prior work: (1) it is, to the authors' knowledge, the first to apply the complete five-stage NEA framework specifically to contextual integer-multiplication word problems, a topic previously examined mainly through misconception inventories rather than stage-by-stage cognitive diagnosis; (2) it combines written-test evidence with in-depth interview probing and observation notes, triangulating three sources to pinpoint the exact stage at which each error originates rather than inferring it from the written product alone; and (3) it generates a stage-specific diagnostic profile for an under-researched region (Lembata Regency, East Nusa Tenggara) under the Indonesian *Kurikulum Merdeka*, yielding remediation guidance targeted to particular cognitive stages instead of generic intervention.

Given the importance of understanding integer multiplication, the systematic nature of cognitive errors, and the multiple factors influencing mathematics learning outcomes, Newman's Error Analysis (NEA) is regarded as the most relevant framework for this study. Through this method, a structured and in-depth understanding of students' errors can be obtained, which can subsequently inform the design of more effective teaching strategies. Based on the above background, the research question is: How do students make errors when solving word problems on integer multiplication, as analyzed using Newman's Error Analysis (NEA)?

II. Research Method

This study involved eleven eighth-grade students from a public junior high school in Lembata Regency, East Nusa Tenggara Province, Indonesia. The sample was purposively selected based on students' semester grades, representing a range of mathematical abilities: high, medium, and low. This sampling strategy ensured that each level of mathematical proficiency was proportionally represented, enabling the researcher to explore variations in student errors across different levels of competence. In addition to academic criteria, students selected were those actively engaged in learning and willing to participate in follow-up interviews.

This study employed a qualitative case study design to enable in-depth exploration of students' errors in solving integer multiplication word problems using Newman's Error Analysis (NEA). The research procedure began with the development of instruments, including a test blueprint aligned with relevant competencies and indicators. A written test comprising ten-word problems involving integer multiplication was constructed. An interview guide was also developed based on the five cognitive stages outlined in NEA: reading, comprehension, transformation, process skills, and encoding.

The test blueprint mapped each of the ten items to a specific content indicator of integer multiplication embedded in a real-life context. Items 1, 9, and 10 measured repeated reduction (negative multiplication as repeated subtraction); items 2 and 4 measured multiplication combined with a secondary operation (loss/additional cost per unit); items 3, 5, and 8 measured rate of change over time (constant increase or decrease across a number of periods); and items 6 and 7 measured aggregated gain or loss (total discount and net score). Each item, therefore, targeted students' ability to translate a verbal situation into an integer-multiplication model and to execute and correctly express the result, providing the basis for classifying errors across the five NEA stages.

To ensure instrument quality, validation was carried out by a mathematics education lecturer and an experienced junior high school mathematics teacher. The aspects validated included content, construct, and language. Based on their feedback, some word problems were revised to enhance contextual clarity, although no conceptual deficiencies were identified. Both validators judged the instrument as suitable for use after minor revision: every item was rated relevant or highly relevant on the content-validity check, so that all ten items were retained. The revisions they requested were limited to wording, clarifying the context of items 3 and 8 and rephrasing the question sentence of item 4, rather than changes to the mathematical content, and the interview guide was endorsed without change. The written test was administered in the classroom under the direct supervision of the researcher, with students given ninety minutes to complete the test. On the following day, individual in-depth interviews were conducted to gain further insights into students' problem-solving processes. Each interview lasted approximately five to seven minutes, was audio-recorded, and transcribed verbatim for analysis.

Data was analyzed using a qualitative descriptive approach. Transcripts and written test responses were first reduced and then coded according to the NEA framework. Each identified error was categorized using a structured coding rubric. Triangulation was employed by comparing data from tests, interviews, and observation notes to ensure validity. While the data were being gathered, the researcher kept observation notes on how each student approached the test-noting, for example, hesitation or repeated erasing at the modelling step, counting on fingers, jumping straight to an operation without rereading the question, and stopping as soon as a number was produced without checking it against the context. During triangulation, an error was confirmed only when all three sources lined up: the script displayed it, the interview uncovered the reasoning behind it, and the observation note matched that reasoning. When the set of interview questions did not elicit a clear account, the researcher added follow-up probes-asking the student to reread the question aloud, explain what each number meant, or redo the computation while thinking aloud, so that the data reflected students' reasoning rather than just their final answers. The researcher interpreted the root causes of the errors based on students' explanations and contextual clues. The analysis focused on the eleven carefully selected students, representing a spectrum of mathematical abilities. Their participation provided a broad range of error patterns, contributing to a deeper understanding of the challenges students face when solving integer multiplication problems.

The NEA procedure traces back to Newman (1977, 1983), who devised it as a straightforward diagnostic tool for pinpointing the errors students make on mathematical word problems. In Newman's account, reaching a correct solution requires passing in order through five cognitive stages: (1) reading recognizing the words and symbols in the problem; (2) comprehension grasping what the problem means in context; (3) transformation recasting the word problem as a fitting mathematical model; (4) process skills executing the necessary calculations or procedures; and (5) encoding and writing down the final answer accurately.

The written test instrument is presented in Table 1, and the interview instrument in Table 2, both developed using an analytical framework based on Newman's Error Analysis (NEA) (Abdullah et al., 2015; Ahmadi et al., 2019; Mubarokah & Amir, 2024):

Table 1. Written test instrument

1	A housewife plans to reduce her family's dining-out expenses by Rp 150,000 each week for 4 weeks. If this expense is reduced consistently each week, what are the total savings the housewife makes over 4 weeks?
2	A vendor purchases 12 boxes of fruit at a price of Rp 30,000 per box. Due to poor quality, he is forced to sell them at Rp 25,000 per box, resulting in a loss of Rp 5,000 per box. What is the total loss incurred by the vendor?
3	A train travels at a speed of 100 km/h. However, due to track resistance, its speed decreases by 5 km/h every hour. What is the train's speed after 6 hours of travel?
4	A customer buys 6 items at Rp 30,000 each. The store offers a discount of Rp 5,000 per item. However, due to a system error, an additional charge of Rp 2,000 is applied to each item. What is the total additional cost the customer must pay?
5	The temperature in City A is recorded at 15°C in the morning. Over the next 4 hours, the temperature increases by 3°C each hour due to hot weather conditions. What is the temperature after 4 hours?

6	In a store, every item purchased at Rp 50,000 comes with a Rp 10,000 discount. If a customer purchases 6 items, what is the total discount they receive?
7	A game player earns 10 points for each level win but loses 5 points for each level lost. After 4 wins and 2 losses, how many points does the player have in total?
8	A store is open daily, but for 3 consecutive days, its opening hours are reduced by 2 hours each day due to technical issues. If the store originally operated for 8 hours a day, how many hours is it open after 3 days?
9	A warehouse manager has 100 boxes of goods, and each week he loses 5 boxes due to damage or unsold stock. If this loss continues for 4 consecutive weeks, how many boxes are lost during that period?
10	A company's stock price drops by Rp 2,000 each day due to unfavorable market conditions. If the price decreases for 5 consecutive days, what is the total decline in stock price over the period?

Table 2. Interview instrument

Type of Error	Questions
Reading	• Please read this problem out loud for me. • Did you come across any words here that were hard to read or unfamiliar to you? • Are these terms ones you have encountered before, and if so, where?
Comprehension	• In your own words, what is this problem actually about? • What do you believe you are being asked to find here? • What makes you confident that that is what the problem means?
Transformation	• Walk me through the approach you took to answer it. • How did you turn this situation into a mathematical statement? • How sure are you about your opening step, and why?
Process Skills	• Take me through how you worked this out. • Could you talk to me through your steps one at a time? • What led you to pick that particular method?
Encoding	• What answer did you settle on, and how do you know it is right? • How did you record your final result? • Did you go back and verify it, and if so, what did you look at?

The instruments outlined above were applied in the interview phase, which followed the written test. Work began by drafting a test blueprint, which then guided the writing of both the test items and the interview prompts. After the blueprint and instruments were finalized, the next step centered on expert validation, conducted to confirm that the items were content-relevant and sufficiently clear to measure students' mathematical ability. Acting on the validators' feedback, the researcher made the needed wording revisions to sharpen contextual clarity before the instruments were used for data collection.

The main data came from a written test specifically designed to elicit the different error types that arise in mathematical problem-solving. Once students had completed the test, the researcher held in-depth interviews with selected participants to probe the thinking behind their written answers and to pinpoint error types using Newman's procedure. These interviews were transcribed and examined qualitatively to surface recurring patterns of error.

In the following stage, each error was sorted into one of the NEA categories. This sorting underpinned the conclusions drawn about which error types predominated and what lay behind them. The researcher then drew the findings together into a final report covering the principal results, their pedagogical implications, and recommendations for strengthening the teaching and learning of mathematics. Taken as a whole, the study set out to build a thorough picture of students' mathematical errors and, in doing so, to support the development of more effective mathematics teaching.

III. Results and Discussion

The analysis showed that students committed a range of errors when solving integer multiplication problems, which were categorized using Newman's Error Analysis (NEA) framework. Five categories were considered: reading, comprehension, transformation, process-skills, and encoding errors. No reading errors were observed among the participants, but the other four categories appeared to differ to varying degrees.

Comprehension Error

The first category identified was the comprehension error, which arises when a student manages to read the problem yet fails to capture its meaning or the contextual details it contains. An error of this sort points to a failure in making sense of the situation the problem describes which in turn obstructs the building of a suitable mathematical model. Figure 1 offers an example: it shows a student's written response to Test Item Number 2, together with an excerpt from the follow-up interview:

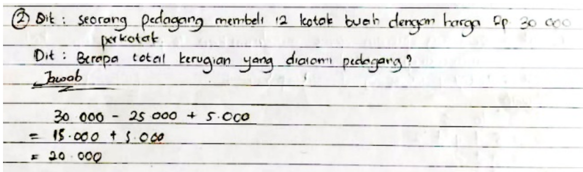

Figure 1. Example of a comprehension error

Test item 2:
A vendor purchases 12 boxes of fruit at Rp 30,000 per box. Due to poor quality, he is forced to sell them at Rp 25,000 per box, resulting in a loss of Rp 5,000 per box. What is the total loss incurred by the vendor?
The following is a student interview transcript:

Teacher : Please read this problem out loud for me.
Student : Yes, Sir. (The student reads the problem as instructed.)
Teacher : Can you explain the question in your own words?
Student : Thirty thousand minus twenty-five thousand, then plus five thousand.
Teacher : Why has it become fifteen thousand plus five thousand?
Student : I thought thirty thousand minus twenty-five thousand was fifteen thousand.
Teacher : Okay, thank you.
Student : Thank you, too, teacher.

In addition to the previous example, a comprehension error is also evident in Figure 2, which shows a student's written response to Test Item Number 3, along with an excerpt from the post-test interview.

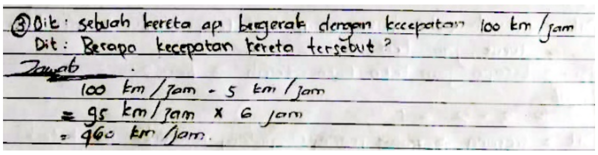

Figure 2. Example of a comprehension error

Test item 3:
A train is moving at a speed of 100 km/h. However, due to track resistance, the speed decreases by 5 km/h every hour. After 6 hours of travel, what is the train's speed?
The following is a student interview transcript:

Teacher: Please read this problem out loud for me.
Student : Sure, Sir. (Student reads as instructed.)
Teacher: Could you explain the question in your own words?
Student : One hundred kilometres per hour minus five kilometres per hour, and then I multiply the result by six.
Teacher: Why did you subtract 5 km/h from 100 km/h and then multiply it by six?
Student : I thought it was because the train traveled for six hours, Sir.
Teacher: Okay, thank you.
Student : Thank you too, Sir.

Based on the students' written work in Figures 1 and 2, along with the corresponding interview responses, it is evident that while students were able to read the questions fluently, they did not fully comprehend the problem statements or the appropriate steps for solving integer multiplication tasks. This suggests a lack of conceptual understanding, particularly in interpreting mathematical contexts involving arithmetic operations. Looking more closely, in Figure 1, the slip sits in the line the student wrote, "30,000 – 25,000 = 15,000", and the interview backs this up ("I thought thirty thousand minus

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twenty-five thousand is fifteen thousand"). What went wrong is interpretive rather than arithmetic: the student read the Rp 5,000 loss per box as a one-off subtraction and missed that the question wants the loss over all twelve boxes (12 × 5,000); the repeated, multiplicative nature of the situation simply went unnoticed. Figure 2 tells a similar story at the step "(100 – 5) × 6". The interview remark ("I thought it was because the train travelled for six hours") reveals that the student followed the storyline but mishandled how the quantities relate—subtracting just once and then scaling the whole figure by the number of hours, instead of seeing that the 5 km/h drop repeats each hour. In each case, the trouble arises while turning the meaning of the situation into a numerical relationship; that is precisely why these are comprehension rather than transformation errors, the failure is in working out what the problem asks before any equation is set down.

Transformation Error

The second category is the transformation error, which occurs when a student grasps the basic sense of the word problem but cannot translate that context into a correct mathematical model. Put differently, the student knows what the task asks yet cannot construct the mathematical representation needed to solve it. Figure 3 illustrates this, presenting a student's written response to Test Item Number 3 (the same item used earlier in the comprehension-error example):

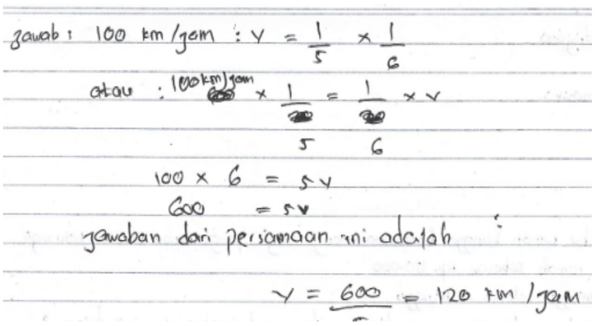

Figure 3. Example of a transformation error

Test item 3:
A train is moving at a speed of 100 km/h. However, due to track resistance, the speed decreases by 5 km/h every hour. After 6 hours of travel, what is the train's speed?
The following is a student interview transcript:

Teacher : Please read this problem out loud for me.
Student : Sure, Sir. (The student reads the question as instructed.)
Teacher : Can you explain the question in your own words?
Student : The train travels at 100 km/h, and its speed decreases by 5 km/h. After 6 hours of travel, the question asks what the speed of the train is.
Teacher : Show me how you solved this problem.
Student : I wrote $100 \times \frac{1}{5} = \frac{1}{6} \times v$, and then just multiplied.
Teacher : Why did you write $100 \times \frac{1}{5} = \frac{1}{6} \times v$?
Student : Because we want to calculate the speed, Sir.
Teacher : Okay. Thank you.
Student : Yes.

Based on Figure 3, while the student was able to read and comprehend the problem, they struggled to transform it into the appropriate mathematical model, which should have been expressed as the linear equation: $V_t = V_0 - (5t)$. Instead, students frequently applied incorrect operations, such as introducing fractions or unrelated algebraic expressions, rather than using integer-based subtraction and multiplication. The interview responses confirmed that the student had a general understanding of the problem's context but lacked the representational fluency necessary to model the scenario mathematically. This aligns with what Newman categorizes as a transformation error, in which the breakdown occurs during the conversion of a contextual narrative into symbolic mathematical form. The error here centers on the line of text "100 × 15 = 16 × v". In the interview, the student recounted the situation accurately ("the train travels at 100 km/h and its speed decreases by 5 km/h… after 6 hours"), so comprehension is not the issue; the problem is that the symbolic form bears no structural link to that account. Figures 15 and 16 match nothing in the problem, and an unknown v is introduced even though the task calls only for repeated subtraction at a fixed rate. This is a representation failure where the student understood the meaning yet could not render "a 5 km/h drop each hour over 6 hours" as 100 – (5 × 6). The student's sole justification, "because we want to calculate the speed," suggests the likely cause: reaching for a remembered, all-purpose equation template instead of assembling the model from the numbers actually given.

Process Skill Error

The third category, alongside the previous two, is the process-skills error. It arises when a student understands the problem and models it correctly but slips computationally or procedurally while working toward the solution. Errors of this kind reveal shortcomings in procedural fluency rather than in conceptual grasp.

An example of a process skill error is shown in Figure 4, which corresponds to Test Item Number 10, along with the results of the follow-up interview:

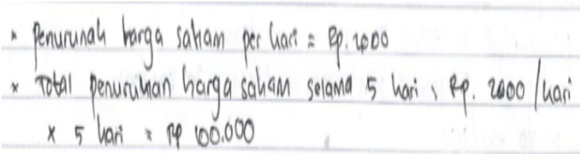

Figure 4. Example of a process skill error

Test Item 10:
A company experiences a decline in its stock price of Rp 2,000 each day due to unfavorable market conditions. If the price continues to fall for 5 consecutive days, what is the total decline in the stock price over that period?
The following is a student interview transcript:

Teacher : Please read this problem out loud for me.
Student : Sure, Sir. (Student reads as instructed)
Teacher : Can you retell the problem in your own words?
Student : The stock price dropped Rp 2,000 per day. The drop continued for 5 days in a row.
Teacher : Show me how you solved this problem.
Student : I wrote 5 days × 2,000.
Teacher : And then?
Student : The result was 100,000.
Teacher : Okay. Thank you.
Student : Yes, Sir.

Based on Figure 4, the student demonstrated an adequate understanding of the problem and constructed an appropriate mathematical model but made an error during the computational process. The slip appears in the student's calculation: "5 × 2,000 = 100,000". The expression "5 days × 2,000" is itself the correct model for a Rp 2,000 daily decline over five days, and the interview confirms that the student read the situation correctly ("the stock price dropped Rp 2,000 per day… for 5 days"); neither comprehension nor transformation is to blame. The only fault is in carrying out the multiplication: 5 × 2,000 equals 10,000, not 100,000. An extra zero has crept into a place-value error in executing the product. Since the breakdown comes after a sound model is in place and lies entirely in the computation, it counts as a process-skills error rather than a conceptual one, signalling shaky procedural fluency when multiplying by multiples of ten.

Encoding Error

The fourth category is the encoding error, which arises when a student performs the required calculations correctly but fails to state the final answer accurately or in the form the problem requires. Figure 5 illustrates this case, drawing on Item 2 of the written test and supported by the follow-up interview.

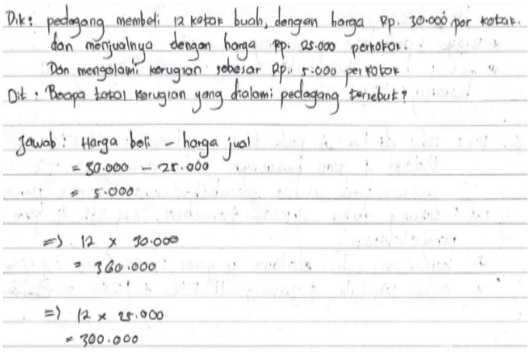

Figure 5. Encoding error

As noted, the encoding error occurs when a student performs the calculations correctly but cannot express the final answer accurately or in line with the problem's requirements. Figure 5 demonstrates this, drawing on Test Item 2, which reads as follows:
A vendor purchases 12 boxes of fruit at a price of Rp 30,000 per box. Due to the poor quality of the fruit, the vendor is forced to sell them at Rp 25,000 per box, resulting in a loss of Rp 5,000 per box. What is the total loss incurred by the vendor?
The following is student interview transcript:

Teacher : Please read this problem out loud for me.
Student : Sure, Sir. (Student reads as instructed)
Teacher : Can you explain in your own words what this question is about?
Student : Twelve boxes are multiplied by 30,000, and twelve boxes are multiplied by 25,000. We are trying to find the loss experienced by the seller.
Teacher : Show me how you solved this problem.
Student : 30,000 minus 25,000 equals 5,000, then 12 × 30,000 equals 360,000, and 12 × 25,000 equals 300,000.
Teacher : What did you do next?
Student : So, the loss is 300,000.
Teacher : Okay, thank you.
Student : You're welcome, Sir.

Figure 5 shows a student who performed the arithmetic correctly yet did not produce a correct final answer, whether in writing or in the interview. In other words, the operations were performed accurately, but the answer was not stated precisely, fully, or as the question demanded. The lapse comes at the closing step; the student puts it into words as "so the loss is 300,000." Everything before it is right. The student found 12 × 30,000 = 360,000 (total cost) and 12 × 25,000 = 300,000 (total sales), evidence of solid process skills, but then offered one of those intermediate figures, 300,000, as the answer rather than the loss. The answer sought is the gap between them, 360,000 – 300,000 = 60,000 (the same as 12 × 5,000). The fault therefore lies not in computation but in encoding: the student stopped short of tying the computed numbers back to what was actually asked, the total loss, and supplied a number that does not answer the question. The interview bears this out, since the student could repeat the correct sub-results yet still called 300,000 the loss, showing the breakdown is in stating and checking the final answer against the problem rather than in the calculation. Table 2 and Figure 6 below summarize students' difficulties using Newman's Error Analysis, drawing on the written test and the follow-up interviews.

Table 2. Summary of error types based on Newman's error analysis

Types of Errors	Student											Number of Errors	Percentage (%)
	1	2	3	4	5	6	7	8	9	10	11		
Reading	0	0	0	0	0	0	0	0	0	0	0	0	0
Comprehension	2	1	0	6	1	7	10	0	0	0	0	27	10.51
Transformation	5	10	2	5	1	10	10	10	2	10	3	68	26.46
Process Skills	5	10	4	4	4	10	7	10	5	10	5	74	28.79
Encoding	5	10	5	9	7	10	10	10	6	10	6	88	34.24

Total	**17**	**31**	**11**	**24**	**13**	**37**	**37**	**30**	**13**	**30**	**14**	**257**	**100%**

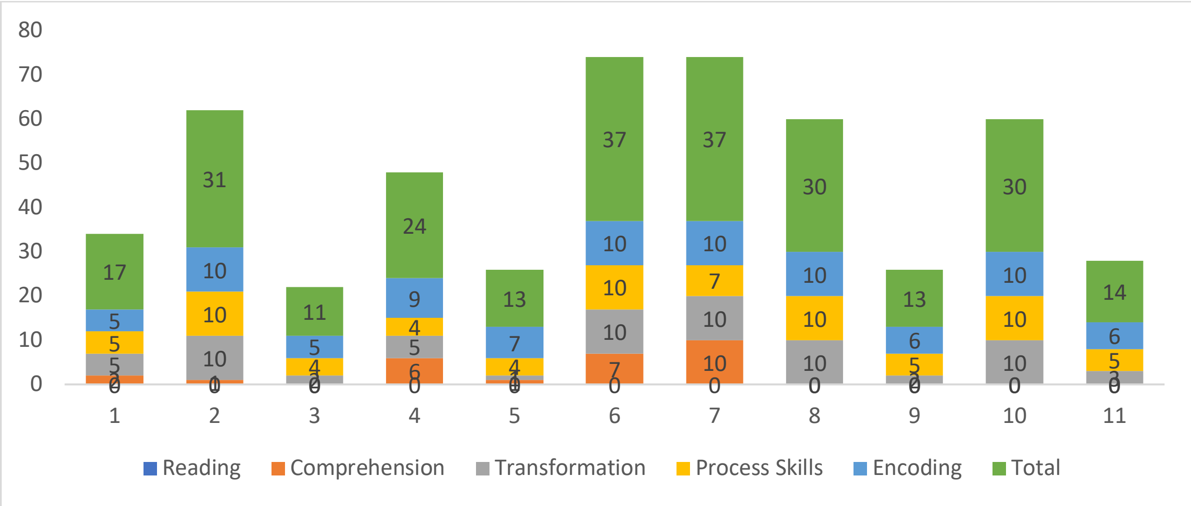

Figure 6. Diagram of the results from Newman's error analysis

Out of a total of 257 student errors in solving mathematical problems, analysis using Newman's Error Analysis (NEA) identified four categories of errors: comprehension (10.51%), transformation (26.46%), process skills (28.79%), and encoding (34.24%). No reading errors were found, indicating that students possess adequate basic literacy skills. Thus, the primary difficulties arise in the stages of interpreting the problem, mathematical modelling, calculation, and presenting the final answer.

These categories were arrived at by triangulating the three data sources. For each error noted above, the written test, the interview transcript, and the researcher’s observation notes were cross-checked, and a case was logged only when all three pointed the same way. The four worked examples make this convergence visible: in everyone (Figures 1–5), the script displayed the error, the interview exposed the reasoning behind it, and the observation notes, such as writing an operation without rereading the question, or halting as soon as a number appeared, fitted that reasoning. Concordance among the sources was generally strong. In the handful of cases where the interview at first clashed with the written work, say, a script that looked like a computational slip but, on probing, turned out to rest on a misread question, the error was reassigned to the stage the converging evidence indicated (in that example, comprehension rather than process skills). Triangulating in this way gives greater assurance that the stage attached to each error captures where the student’s reasoning genuinely faltered, not merely what the written answer suggested.

At 34.24%, encoding was the leading error type, indicating that students often stumbled when writing their final answers, reporting incorrect results, giving responses that did not fit the context, or presenting answers in an illogical way. This echoes Nofrianto, Gulo, Amri, & Rafulta (2022), who drew attention to students' trouble in forming sound conclusions from their problem-solving work.

Process-skills errors accounted for 28.79%, arising from missteps in the procedure itself, such as hand-calculation errors or misapplied formulas. Transformation errors accounted for 26.46%, indicating that students, especially lower-achieving ones, found it hard to construct appropriate mathematical models from verbal information, a point stressed by Arifin, Zulkardi, Ilma, & Hartono (2021). Comprehension errors were comparatively fewer at 10.51% yet remain important because they bear on how students first make sense of the problem; this is in line with Suradi & Djam'an (2021).

Lubis & Jupri (2023) offer a

counterpoint to most prior work, finding that students' comprehension was not a significant predictor of their success with word problems. That counters to many studies, including the present one, in which comprehension is treated as a foundational NEA stage. The implication is that, although NEA reliably surfaces errors along its sequence from comprehension through to encoding, its stages may not carry equal weight for every kind of error; comprehension, on Lubis and Jupri's evidence, does not always drive performance. NEA therefore remains a useful diagnostic instrument, but one best paired with complementary techniques, think-aloud protocols or real-time cognitive observation, say, to capture the moving, dynamic quality of students' reasoning.

Beyond simply locating the errors, the overall pattern can be read through the lens of well-established theory. That errors cluster at the transformation and process stages fits the long-recognized divide between conceptual and procedural knowledge (Hiebert & Lefevre, 1986; Rittle-Johnson, Schneider, & Star, 2015): students who run multiplication procedures without a connected sense of what the operation means tend to lean on memorized routines that collapse once the problem appears in an unfamiliar guise. The frequency of transformation errors, in particular, reflects trouble shuttling between the verbal, situational form of a problem and its symbolic form, the translation step at the heart of Mayer's (1992) model of mathematical problem solving, where representing the problem must come before working it. Seen this way, an encoding error is not a mere careless slip but a failure to tie the computed result back to the situation, betraying weak metacognitive monitoring of whether the answer makes sense in context. The reading also aligns with cognitive load theory (Sweller, Van Merriënboer, & Paas, 2019): when students lack automated, schema-based routines for integer multiplication, working memory is consumed by low-level computation, leaving little for the higher-order checking needed to verify and express the answer properly. Read against these theories, the high rate of encoding errors here suggests that teaching has fostered procedural execution more than the building of durable mental representations and self-regulatory checking. The NEA findings thus do more than flag where errors lie; they identify the specific mechanisms representation, schema automation, and metacognitive verification, that remediation ought to target.

Viewed through the lens of information processing, the four error stages mark successive points at which cognition can break down. That no reading errors occurred tells us decoding the text was not the obstacle; trouble set in once meaning had to be assembled. The comprehension errors discussed earlier, for instance, reading a per-unit loss as a single subtraction in Figure 1, point to a failure to construct an adequate situation model, the internal representation that, in Mayer's (1992) account, has to be in place before any solution plan can be formed. The transformation errors, such as the disconnected equation "100 × 15 = 16 × v" in Figure 3, reflect a faulty mapping of that situation model onto a mathematical structure, in keeping with thin schema availability for integer multiplication: without a schema to draw on, students reach for generic equation templates. The process-skills error, the place-value slip "5 × 2,000 = 100,000" in Figure 4, is most readily explained by cognitive load: when basic facts are not automatic, working memory is overtaxed, and slips creep in. Lastly, the predominance of encoding errors (Figure 5, where an intermediate value is offered as the final loss) indicates weak metacognitive control. Students did not check whether their answer actually responded to the question. Read against these construction models, schema construction, working-memory load, and metacognitive monitoring. The results explain not just where the errors happened but why, and they suggest that remediation should cultivate conceptual representations and habits of self-checking rather than rely on procedural drill alone.

In practical implementation, teachers

should focus not only on strengthening students' reading comprehension but also on improving their ability to transform problems, apply procedures correctly, and most importantly, communicate their mathematical thinking effectively. These efforts are essential to minimize encoding errors and improve overall problem-solving outcomes.

Several limitations should be kept in mind when reading these results. The foremost is the very small sample, just eleven students (n=11), which limits the statistical power of the findings and constrains how far they can be generalized. The results, in short, cannot be transferred directly to a wider student population, whether geographically, socially, or academically.

The study was also set in a single, particular context, one school in Lembata Regency, which further narrows the reach of the findings. The school’s distinctive features, the students’ backgrounds, and the local learning conditions may all have shaped the results. Therefore, the findings may not represent the conditions or responses of students from other areas or schools with different profiles.

A further limitation concerns the test items themselves. The instrument did not give even coverage to all arithmetic operations: addition, subtraction, and division among them, and this uneven coverage may have skewed the assessment of students’ overall numeracy. Abilities in the under-represented operations may thus have gone insufficiently measured, leaving the analysis less complete than it might otherwise be.

Given these limitations, the interpretation of the research findings should be approached with caution. Future studies are recommended to involve larger and more diverse samples, including schools from various regions, and to use more balanced instruments that cover the full range of mathematical operations to obtain a more comprehensive understanding of students' numeracy abilities

IV. Conclusion

Examining the errors eighth-grade students in Lembata Regency make on integer multiplication problems yields more than a set of percentages; it exposes important gaps in their mathematical thinking that call for responsive, well-targeted teaching. Reading errors were almost absent, yet the sizeable rates at comprehension (10.51%), transformation (26.46%), process skills (28.79%), and encoding (34.24%) show how many-sided the difficulty is spanning understanding, modelling, procedural fluency, and the presentation of answers.

That encoding drew the largest share of errors. Students computed correctly but failed to answer the problem actually called for, points to a real gap in their working habits. It suggests they are not in the habit of reviewing or verifying their answers, a skill central to mathematical competence. Interventions, then, should go beyond delivering content to deliberately nurturing self-monitoring and reflection: teachers can build in regular routines for error-checking and self-correction so that students grow more careful in finishing and revisiting their work.

Transformation and process errors are equally pressing, stemming from limited conceptual understanding and weak procedural fluency. This implies that many students execute memorized procedures without grasping their purposes or how to adapt them to varying contexts. Addressing this requires conceptually rich instructional strategies that build both conceptual and procedural knowledge.

Errors at the comprehension stage likewise underline how much students struggle to interpret the information in word problems, pointing to a need for explicit teaching of mathematical literacy. Teachers can help students unpack contextual problems and translate them into mathematical language, through critical-thinking tasks, group discussion, or visual mapping, for instance.

Overall, these findings underscore the strength of Newman's Error Analysis as a diagnostic framework that not only identifies the types of student errors but also explains why students struggle. Consequently, instructional strategies should not be generic but aligned with the four error components. Comprehension, emphasize contextual reading strategies and problem interpretation; transformation, strengthen conceptual explanations through guided visual modelling and abstraction; process, develop procedural fluency via structured practice and varied problem types; and encoding, foster reflective practice and habits of answer verification.

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