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Design research on developing primary students' conceptual understanding of area through visual-spatial activities

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Abstract
This study examines how primary students develop a conceptual understanding of composite shapes, a topic that many learners find challenging beyond the routine use of formulas. The research was motivated by the persistent issue that instruction in the area in elementary classrooms is often procedural and does not sufficiently foster reasoning about spatial structure, decomposition, and measurement. This study employed a Design Research approach in a sixth-grade classroom (n = 6 students) to iteratively develop and test a Hypothetical Learning Trajectory (HLT) for the learning area of composite shapes through visual-spatial and puzzle-based activities. Data were collected through a pre-test, two instructional activities (Activity 1 and Activity 2), classroom observations, and a post-test. Data were analyzed qualitatively through retrospective analysis aligned, with HLT, supported by descriptive coding of students' strategies, triangulation of written work, and teacher field notes. Findings indicate that students shifted from purely visual recognition toward more analytical strategies, such as partitioning, conservation, transitivity, and additivity, which emerged during instruction. The results suggest that structured visual-spatial tasks grounded in a realistic context effectively bridge abstract area concepts and classroom practice, informing future instructional design in elementary geometry.

Keywords: area composite shapes; spatial reasoning; area concepts; HLT; design research

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I. Introduction

Understanding the area of composite shapes is a fundamental yet challenging topic for primary school students. Unlike simple plane figures, composite shapes require students to identify constituent shapes, apply appropriate formulas, and integrate results within a meaningful context. Research has shown that many students fail to develop a deep understanding of area because instruction tends to emphasize rote memorization over conceptual reasoning (Ilmi, Maulana, & Sudin, 2021; Utami, Hartono, & Putri, 2022). Consequently, students often struggle to interpret measurement situations, particularly when faced with compound figures or real-world applications.

In many Indonesian elementary classrooms, lessons are teacher-centered, focusing on procedural demonstration rather than exploration or justification. Students can recite formulas for rectangles or triangles but have trouble when they must decompose irregular figures, infer missing side lengths, or justify their strategy. These patterns suggest that learning environments rarely engage students in visual–spatial reasoning or discussions that connect formulas to the meaning of area itself. As a result, they acquire procedural familiarity without robust conceptual grounding in spatial structure.

To address this issue, mathematics educators have increasingly emphasized the importance of building conceptual foundations in measurement. Understanding area involves four essential concepts: conservation**,** transitivity**,** unit iteration**,** and additivity. Conservation refers to the understanding that the area of a shape remains constant despite rearrangements, as long as no material is added or removed. Transitivity allows students to compare areas indirectly—for instance, if Shape A is larger than Shape B, and B is larger than Shape C, then A must be larger than C. Unit iteration refers to the process of covering a surface with repeated units without gaps or overlaps. Finally, additivity emphasizes that the area of a complex shape is the sum of the areas of its non-overlapping components. Clements & Sarama (2004) argue that these four concepts form the basis for meaningful understanding of an area and should be consistently embedded within students' learning experiences.

Research on spatial visualization also highlights that students' strategies in determining area are strongly influenced by their ability to mentally manipulate, decompose, and realign shapes (Patahuddin, Logan, & Ramful, 2018; Patahuddin, Ramful, Lowrie & Bholoa, 2022). Such visual–spatial manoeuvres enable learners to perceive composite figures as combinations of simpler familiar shapes, supporting deeper conceptual understanding rather than procedural calculation alone (Xu, Sun, & Kong, 2025). A recent international study also emphasizes that spatial ability development plays a crucial role in supporting conceptual understanding in mathematics learning contexts (Zhu et. Al., 2023). These foundational ideas highlight the need for instructional approaches that foreground reasoning rather than formulaic practice. This perspective aligns with research showing that students' spatial visualization strategies when working with composite shapes involve decomposition and recomposition processes (Patahuddin, Logan, & Ramful, 2018).

Recent studies emphasize that visual–spatial activities, particularly puzzle-based tasks, play a crucial role in helping students build conceptual understanding of area through meaningful manipulation and reconfiguration of shapes (Ekawati, Kohar, Imah, Amin & Fiangga, 2019; Herrera, Ordonez, & Loza, 2024; Sari, Fiangga, Milla & Puspaningtyas, 2023). Puzzle activities support students' ability to conserve area during cut-and-paste processes and enhance their252easonning when dealing with composite shapes, making them highly relevant to primary mathematics learning.

In the context of the Indonesian curriculum (Kurikulum Merdeka as well as the previous 2013 Curriculum), learning about area at the elementary level aims not only for procedural fluency in using formulas but also for conceptual understanding of measurement and spatial reasoning. Students are expected to explore how units compose and decompose surfaces, compare different shapes through reasoning, and connect measurement to real-life contexts. Therefore, integrating Stephen and Clements' framework into instructional aligns closely with national learning objectives that emphasize reasoning, presentation, and problem solving rather than mere formula practice.

However, classroom practice tends not to foreground these four concepts. Teachers frequently move directly to "apply the formula," rather than supporting children in structuring space, reasoning about sub-regions, and articulating why two differently arranged shapes can still have the same area. As a result, students may succeed on routine textbook items but struggle to reason about unfamiliar or realistic composite figures (Midianti & Zainil, 2021). In response to these pedagogical shortcomings, researchers and practitioners have turned to more context-oriented instructional approaches.

Unfortunately, conventional teaching methods rarely prioritize these principles. The dominant practice in many classrooms involves giving students formulas and procedural tasks, with limited exploration or contextualization. Classroom observations and prior studies in Indonesia confirm that such practices remain prevalent, where teaching tends to prioritize procedural fluency over conceptual understanding (Ilmi et al., 2021; Utami et al., 2022). As a result, students are unable to transfer their knowledge to unfamiliar problems or recognize the real-life value of measurement skills. This condition has led to the adoption of innovative pedagogical approaches such as Indonesian Realistic Mathematics Education (PMRI). PMRI is adapted from the Dutch RME theory, developed by Freudenthal, which emphasizes that mathematics should be experienced as a human activity rooted in real-world contexts (Zulkardi, Putri, & Wijaya, 2020).

PMRI promotes progressive formalization of mathematical ideas, beginning with contextual problems and gradually introducing abstract symbols. Its learning principles include the use of meaningful contexts, student contributions, model development, interactivity, and horizontal mathematization. When applied to the teaching area, PMRI encourages students to explore spatial relationships, estimate measures, and reason through decomposition strategies using real-life examples. For instance, they might measure parts of a school garden, a house layout, or tiles on a classroom floor—contexts that make the concept of area more tangible and relevant (Midianti & Zainil, 2021).

To strengthen these conceptual processes, puzzle-based activities have emerged as an effective instructional medium. Indrawati, Arjudin, & Fauzi (2023) demonstrated that puzzle-based learning provides a physical and visual representation of mathematical relationships, allowing students to manipulate, rotate, and decompose shapes directly. Her study showed that puzzles enhance students' spatial reasoning and facilitate a deeper understanding of mathematical concepts such as conservation and additivity. Through puzzle tasks, students practice unit iteration and observe how rearranged pieces still occupy the same area, fostering a deeper grasp of measurement principles.

Moreover, puzzle-based activities promote active engagement, collaboration, and problem-solving, all of which align with PMRI principles. Recent international research has shown that incorporating manipulative and puzzle-based media into geometry learning significantly improves students' motivation, engagement, and conceptual understanding of shapes (Ponte, Viseu, Neto, & Aires, 2023). Similarly, Nurwahyuningsih, Cahyadi, & Suyitno (2024) reported that puzzle-based instruction enhanced both conceptual understanding and procedural fluency in measuring composite shapes. Together, PMRI and puzzle-based learning directly address the difficulties identified earlier—helping students move beyond memorizing formulas toward constructing relational understanding of area in composite shapes through contextual exploration and hands-on decomposition. These findings suggest that combining PMRI with puzzle media creates a powerful pedagogical synergy for supporting the mathematical development of primary students.

To systematically implement and refine such instructional innovations, many researchers employ Design Research methodology. As articulated by Gravemeijer & Cobb (2006), Design Research involves iterative cycles of designing, testing, and revising learning environments based on theoretical and empirical analysis. This approach not only produces effective instructional tools but also contributes to the development of local instructional theory — here, an empirically grounded Hypothetical Learning Trajectory (HLT) for the learning area of composite shapes.

Despite increasing attention to PMRI and puzzle-based learning, Lestari & Rahmawati (2023) noted that prior studies have not always made explicit how students' conceptual understanding of area emerged step-by-step. Similarly, Pamungkas, Wardono, & Agoestanto (2024) emphasized that many learning designs have not clearly illustrated how specific sub-activities support key sub-comncepts such as aconservation and unit iteration. This study addresses that gap by documenting how students' strategies evolved across tasks and by relating those strategies to four foundational area concepts (partitioning, conservation, transitivity, additivity) in the context of composite shapes.

Several recent studies have successfully used Design Research to create PMRI-based instructional sequences. Lestari & Rahmawati (2023), for example, designed learning tools using cultural artefacts and found increased student engagement and spatial awareness. Pamungkas et al. (2024) integrated PMRI with digital applications such as Photomath, revealing improved curiosity and independence among learners. Likewise, Afidah, Wardono, Stefanus, & Waluya (2024) observed that PMRI-based tasks enhanced better mathematical communication and reflective thinking in geometry lessons. These studies underscore the relevance of developing a well-structured HLT that captures the emergence of area concepts in meaningful, context-driven activities.

Several recent international studies incorporating the PMRI or RME approach have successfully utilized puzzle-based and visual–spatial media to promote meaningful mathematical understanding and communication (Herrera et al., 2024; Kyaw & Vidákovich, 2025; Kurniasih & Ngastiti, 2024). The use of physical and digital puzzles enables students to explore mathematical concepts through manipulation and representation, aligning well with PMRI principles of guided reinvention and mathematization (Gorev, Telegina, Karavanova & Feshina, 2018). Empirical evidence also indicates that Realistic Mathematics Education can significantly enhance students' cognitive achievement in topics such as geometry when contextual tasks and meaningful models are used to support problem-solving and reasoning (Laurens, Batlolona, Batlolona & Leasa, 2017).

However, few studies have specifically examined how PMRI-based instructional design, when combined with puzzle-based activities, can foster students' conceptual understanding of areas in composite shapes. This study addresses that gap by focusing on how key area concepts emerge within a structured PMRI-puzzle framework.

Based on these aims, this study was guided by the following research questions. (1) how do primary students' strategies for determining the area and perimeter of composite shapes develop during PMRI-oriented, puzzle-based instructional activities?; (2) in what ways do these strategies reflect the emergence of key area concepts (partitioning, conservation, transitivity, and additivity) envisioned in the Hypothetical Learning Trajectory (HLT).

In line with these questions, this article reports finding from the first design cycle (pilot experiment) and describes how students' reasoning evolved across the implemented activities.

II. Research Method

This study employed a Design Research approach with a Validation Study type to develop a Hypothetical Learning Trajectory (HLT) for the topic of area of composite shapes using puzzle-based activities. This approach was chosen because it aligns with the intention of developing and validating a local instructional theory, as explained by Gravemeijer & Cobb (2006), which aims to understand the progression of students' conceptual understanding within authentic classroom settings. Design research was considered more appropriate then conventional experimental designs because the primary goal was to iteratively design, test, and refine an instructional sequence rather than to test a fixed treatment for large-scale generalization.

The research was conducted at State Primary School 244 Palembang, involving 6 sixth-grade students (aged 11-12 years) selected through purposive sampling based on teacher recommendation to represent a range of mathematical abilities (high, medium, and low).

Although the sample size was a small, it was considered appropriate for a pilot teaching experiment in Design Research, where the primary purpose is to refine learning design than to generalize findings. This small-group setting is consistent with typical DR pilot cycles, which often involve 4-8 students to allow close observation of students' strategies and reasoning (Gravemeijer & Cobb, 2006).

The HLT and instructional sequence were aligned with the Grade 6 mathematics learning outcomes in the Indonesian curriculum, particularly those requiring students to determine and compare the area and perimeter of various plane figures, including composite shapes, using appropriate units and reasoning. The teaching experiment consisted of two classroom meetings (approximately 70 minutes each), both situated within the irregular mathematics lessons on area and perimeter.

The study considered of three main phases:

Preparation phase

Which included designing the HLT and sequencing learning activities that emphasized four core area concepts—partitioning, conservation, transitivity, and additivity. In this phase, visual-spatial and puzzle-based tasks were design to encourage students to cut, rearrange, and compose shapes, so that the four concepts could emerge progressively through contextual PMRI-oriented activities.

Teaching experiment

In which the HLT was implemented in two classroom sessions using puzzle-based PMRI activities. In the first session, students explored composite figures (eg, Gingerbread-shape regions and familiar layouts) by partitioning them into simpler subshapes and investigating whether rearrangements changed the area (partitioning and conservation). In the second session, students inferred unknown side lengths, recomposed subregions. They calculated the area and perimeter of composite shapes (using the principles of additivity and transitivity) with puzzle pieces and concrete supports. Data were collected through classroom observations, video recordings, students' written work, and field notes.

Retrospective analysis

Where the actual learning trajectory was compared with the hypothetical design to evaluate its effectiveness and answer the research questions. The comparison focused on how students' strategies and explanations in each activity reflected the four key concepts (partitioning, conservation, transitivity, and additivity) envisioned in the HLT, and how these conceptual indicators developed across the two meetings. This article reports findings from the first design cycle (pilot experiment).

To evaluate students' conceptual understanding, a pre-test and post-test were administered before and after the teaching experiment. Both instruments were developed based on the four foundational area concepts—partitioning, conservation, transitivity, and additivity—drawing from the framework of area measurement reasoning described by Lehmann (2023) and recent spatial reasoning research by Pinilla, Gilligan-Lee, Hodgkiss, Outhwaite & Simms, (2024), which highlight the importance of progressive conceptual understanding in early geometry learning. Each test consisted of open-ended tasks that required students to explain their reasoning, allowing for a qualitative analysis of their spatial and conceptual development.

Data were analyzed qualitatively and retrospectively, using the HLT as an analytical framework. As suggested by Gravemeijer & Cobb (2006), the analysis involved three main steps: (a) data reduction; (b) thematic coding based on the four concepts, and (c) triangulation across multiple data sources (classroom observations, students' written work, and field notes) to ensure validity and reliability. In particular, we examined changes in the students' strategies and justifications from the pre-test to the post-test and across two lessons, looking for evidence of increasingly sophisticated use of partitioning, conservation, transitivity, and additivity when working with composite shapes. The findings were interpreted descriptively to identify patterns, deviations, and areas for improvement in the learning design. To provide clarity on the analytical lens, the HLT guiding this study is summarize in Table 1.

Table 1. Summary of the HLT

Activity	Learning goals and Hypothesized Student Thinking
Constructing and Partitoning a Gingerbread Shape	**Goal**: To recognize composite figures and partition them into meaningful subregions. Hypothesis: Students explore various ways of cutting, identify subshapes, and realize that rearranging pieces does not change the area. (Concepts: partitioning, conservation.)
Determining Unknown Side Lengths	**Goal**: To infer missing side lengths and apply additive reasoning. **Hypothesis**: Students match corresponding sides, decompose and recombine subregions, and compute area systematically. (**Concepts:** additivity, transitivity)
Expected Outcome	Students can solve composite-area problems in a structured and conceptually grounded manner.

To assess students' initial understanding, two diagnostic pre-tasks were used. Each problem was designed to reveal different aspects of conceptual and spatial reasoning related to area measurement. The two diagnostic tasks designed for this purpose are shown in Figures 1 and 2.

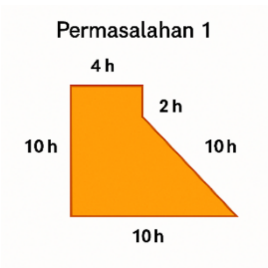


Figure 1. Pre-test item on the area of composite shapes (Problem 1)

(1) problem 1 (Figure 1) involved a composite figure combining a rectangle and a right triangle, aiming to assess students' ability to *partition* shapes and apply *the principle of additivity* in calculating total area; (2) problem 2 (Figure 2) presented a rectangle with a semicircular section removed, focusing on *conservation* and *unit iteration*.

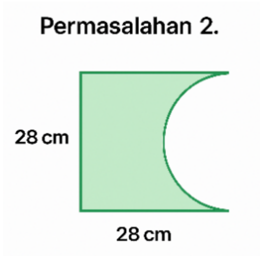


Figure 2. Pre-test item on the area of composite shapes (Problem 2)

These open-ended tasks were analyzed thematically to identify evidence of the four foundation concepts through students' written responses, sketches, and explanations.

III. Results and Discussion

The following section reports findings from the first design cycle (pilot experiment). The stages of this experiment consist of a pre-test, a teaching experiment (comprising Activity 1 and Activity 2), and a post-test. The results indicate that the two main activities designed within the Hypothetical Learning Trajectory (HLT) were effective in supporting students' understanding of the concept of composite shapes.

These findings are consistent with international research, which reports that puzzle-based visual–spatial tasks significantly contribute to the development of area concepts through conservation activities and shape restructuring (Sari & Ng, 2022; Sari et al., 2023). Students' engagement in manipulating and composing shapes supports the emergence of conceptual understanding envisioned in the HLT, confirming the role of visual–spatial reasoning in primary mathematics learning (Patahuddin et al., 2022; Xu et al, 2025). This involves subtle mental manoeuvres in which learners restructure composite shapes through internal visualization processes (Patahuddin, Ramful, Lowrie & Bholoa, 2022).

Further evidence shows that structured manipulatives and movement-based geometry tasks contribute to strengthening young learners' spatial perceptions and foundational geometry understanding (Kılıç & Yorulmaz, 2023), and that spatial ability development more broadly plays a crucial role in supporting conceptual understanding in mathematics learning contexts (Zhu et al., 2023).

Pre-Test

Prior to conducting the pilot experiment, the researcher administered a pre-test to the students. The purpose of this test was to explore the strategies employed by students, which were then discussed collectively through a whole-class discussion. Figures 3 & 4 present an example of a student's response.

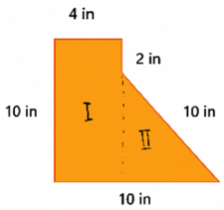


Figure 3. Example of student partition results

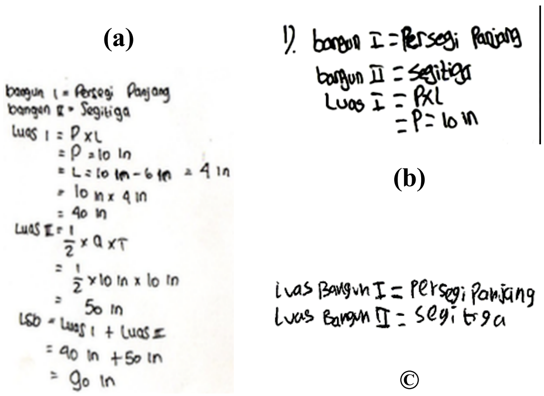


Figure 4. Some of the students' pre-test answers

The pre-test results showed that although students were able to identify the component shapes and write the correct formula, they struggled mainly with procedural aspects such as determining the appropriate base and height or inferring the missing side length. This indicates that their errors were procedural rather than conceptual misconceptions. In terms of the four area concepts, students demonstrated an initial ability to partition, but had not yet coordinated additivity and transitivity. These findings match the entry-level assumption of the HLT, which expects partial conceptual readlines that still require structured support to integrate these ideas when solving composite-area tasks.

Teaching Experiment

Subsequently, the researcher proceeded to the teaching experiment phase, which consisted of two main activities. The **first activity** involved assembling puzzle pieces composed of basic geometric shapes such as rectangles, triangles, and trapezoids to construct a composite figure. Figure 5 below illustrates the flat-shape puzzle used by the students during this activity.

Figure 5. Flat-shape puzzle used in Activity 1

In his first activity, the researcher created four problems. In Problems 1 and 2, students were asked to create new shapes by combining two or three of the provided flat-shape puzzle pieces into a new cake shape. Some students combined the pieces as shown in Figure 6.

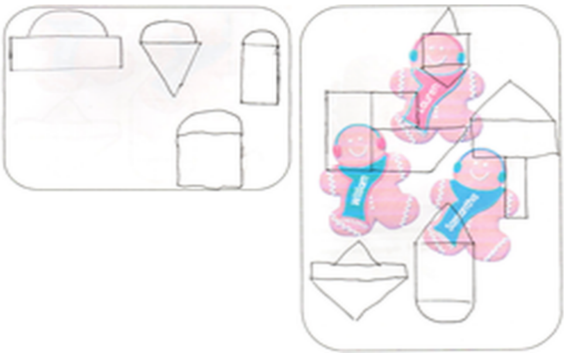


Figure 6. Examples of students' constructed in Activity 1.

Then, in Problem 3, the researcher began to elicit students' understanding of the concept of partitioning by asking them to identify the basic flat shapes that make up a gingerbread cookie shaped like a person. See the student's response in Figure 7.

Figure 7. Students' responses to Problem 3 (identifying component shapes of a gingerbread-like figure)

By using puzzle pieces in the shapes of rectangles, triangles, and trapezoids, students were able to combine 2–3 pieces to form a composite shape. This activity aimed to develop students' visual and spatial abilities in recognizing component shapes, as well as to build their understanding of the concepts of conservation and unit iteration.

In Problem 4, it was observed that students had begun to identify the basic geometric shapes that composed each figure by visually partitioning them. This was evident from the fact that only a few of the figures included additional partitioning lines, indicating that most students were able to mentally decompose the composite shapes without relying on visual aids, as shown in Figure 8.

Figure 6. Students' responses to Problem 4 (visual partitioning of composite figures)

Based on observations, most students were able to assemble composite shapes using a variety of strategies. Some arranged the pieces based on side lengths, while others applied a symmetrical approach. Group discussions encouraged students to communicate their strategies and reflect on their understanding of basic shapes. This process showed that puzzles, as concrete learning media, helped students grasp that changes in position or orientation do not affect area, indicating an emerging understanding of area conservation, consistent with Sari & Ng (2022) who emphasize that concreate and visual-spatial tasks help children coordinate trasnformations without altering the underlying geometric attributes.

Through this activity, students demonstrated their ability to identify component shapes and reorganize them into different arrangements without altering their total size. This reflects an emerging understanding of conservation, where students recognized that rearranging shapes does not change the total area. The task also encouraged the development of students' spatial abilities, particularly in visualizing and manipulating geometric figures. Group discussions during this activity helped strengthen students' reasoning strategies and their ability to communicate mathematical ideas in informal ways.

Based on the activities carried out in Activity 1, students did not encounter significant difficulties in solving Problems 1 and 2. In these two problems, they were required to combine basic flat-shape puzzle pieces to form a new shape resembling gingerbread. However, in Problem 3, students began to experience more difficulty. To solve this problem, students had to explore various possible combinations that could lead to a correct solution. Some students were observed using a trial-and-error approach by physically manipulating the puzzle pieces, while others appeared to pause and engage in visual partitioning mentally before testing their imagined combinations with the actual puzzle pieces. Within the allotted time, no student was able to find the correct combination independently. Consequently, the teacher provided a clue to help guide them toward the correct configuration.

In Problem 4, students started to become more accustomed to solving problems without necessarily drawing partitioning lines on every given shape. Overall, Activity 1 proved helpful in fostering students' visual reasoning skills in mentally decomposing composite shapes.

Overall, the sequence from Problem 1 to Problem 4 shows a gradual shift in students' thinking—from relying on concrete manipulation to beginning to use mental decomposition and implicit conservation. This pattern aligns with the early spatial-structuring stages outlined by Clements & Stephan (2004), where learners move from perceptual actions toward more coordinated visual–spatial reasoning. The evidence indicates that students' conceptual activity is still centered on partitioning and conservation, while additivity and transitivity have not yet appeared, which is consistent with the entry assumptions of the HLT. These results also support previous findings that visual–spatial tasks promote initial conceptual growth but require further scaffolding to develop deeper relational understanding.

The **second activity** consisted of three problems, which were designed to help students identify the unknown side lengths of composite shapes. This step was essential in enabling students to subsequently calculate the perimeter and area of the given composite figures.

Problem 1 consisted of two questions. The first question is illustrated in Figure 9 below.

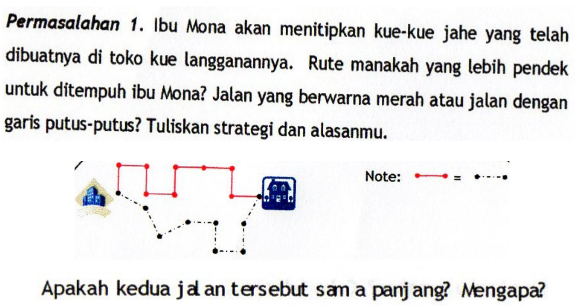
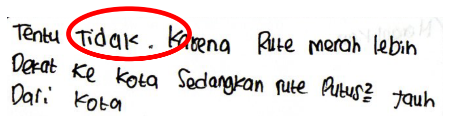


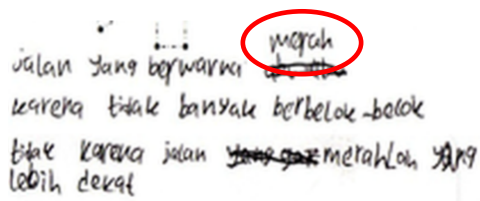
Figure 9. Problem 1 activity 2 (comparing two routes)

In Problem 1, students provided identical answers, and both responses featured sections that had been heavily crossed out. Upon closer examination, it was evident that in both the first and second students' answers, the areas marked with thick strokes corresponded to what was originally labelled as the "grey route." This discrepancy occurred due to a printing error, where the route intended to appear in red was instead printed in grey. In the figure, both groups justified their selection of the red route as the faster option. Their justifications fell under non-measurement reasoning, as they relied primarily on visual judgment rather than actual measurements.

Refer to Figure 10 for students' responses to Problem 1. Both students provided identical answers, with the same portions of their work visibly marked with bold strikethroughs. Upon closer examination, these bold markings—present in both Student 1 and Student 2's responses—indicate that their original answers followed the grey route. This occurred due to a printing error, where the intended red route appeared as grey on the activity sheets. In the figure, both groups justified their final choice of the red route as being the faster one. The reasoning provided by both groups falls under non-measurement reasoning, as it relied primarily on visual judgment rather than numerical or formal measurement.



(a)



(b)

Figure 10. Students' responses to problem 1, activity 2

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The second question in Problem 1 was designed to explore the strategies employed by students. Upon comparing the responses shown in Figure 8, it can be concluded that the students' reasoning about length falls within the category of non-measurement reasoning, specifically at level N0: *Holistic Visual Comparison*. This level, as described by Clements & Stephan (2004), refers to students' reliance on intuitive and visual judgments to compare lengths without engaging in actual measurement or quantitative analysis. The responses indicate that the students determined the faster route solely by visually assessing the paths rather than calculating or estimating numerical distances.

The question in Problem 2 of the second activity was designed to be slightly more complex. This problem also emphasized students' understanding of length. Figure 11 below illustrates the form of Problem 2 along with an example of a student's response.

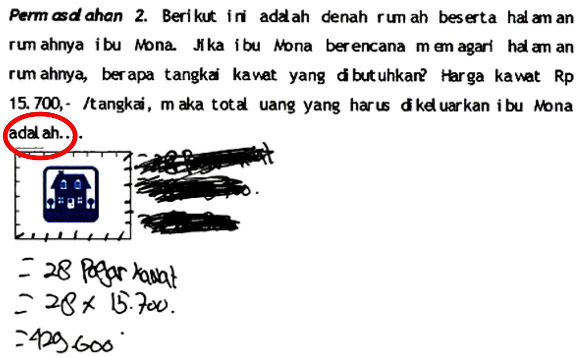
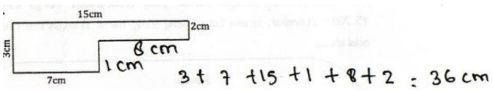


Figure 11. Problem 2 and example student's answer (unit-length iteration along fences)

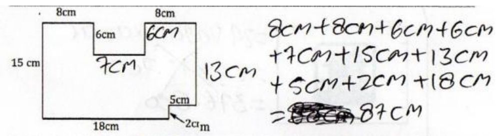
As shown in Figure 11, the students provided incorrect answers. The error occurred because they failed to incorporate additional information embedded in the image. This information—highlighted by the red circle—indicates that one "stem" consists of two small fence sections. None of the students from either group in this first cycle succeeded in answering Problem 2 correctly.

In Problem 3, students were asked to determine the lengths of the missing sides of three composite shapes and then calculate their perimeters. For this final problem in Activity 2, students did not encounter significant difficulties when calculating the unknown side lengths or determining the perimeters of the first and second composite shapes. These two figures were composed of several squares and rectangles, with which students were already familiar.

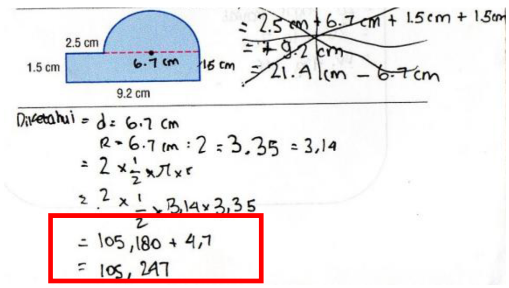
However, students experienced difficulty with the third composite shape. Unlike the previous two, the third figure—when partitioned—consisted of one or two rectangles and a semicircle. The challenge arose because students were not yet accustomed to calculating the perimeter of a semicircle. Figure 12 presents several student responses to Problem 3



(a) Giana's



(b) Anggun's



(c) Rizky's

Figure 12. Students' response to Problem 3 (determining missing lengths and perimeter of composite figures)

The primary focus of Activity 2 was to familiarize students with identifying unknown side lengths in composite shapes. By determining these missing lengths, students were encouraged to decompose the shapes into simpler components, which in turn facilitated their ability to calculate both the area and perimeter of the given figures.

Based on the implementation of this activity, it was evident that students generally understood the essence of each problem. Although several incorrect responses were observed, this did not diminish students' enthusiasm for completing the given tasks. Errors in Problem 1 were partly caused by the unequal structure of the paths presented: the red path consisted solely of horizontal and vertical lines, while the dashed path included a diagonal segment. This diagonal element not only confused the students but also did not encourage meaningful comparison between the two paths. Ideally, the task was designed to promote reasoning strategies such as rearranging parts for direct comparison *and* one-to-one matching of units—processes that Flavin, Chung, Hwang, & Flavin (2025) identified as central to developing students' conceptual understanding of area measurement in interactive and spatially rich learning environments.

Problem 2 had a similar objective, yet with a different emphasis. While Problem 1 aimed to elicit visual and non-measurement reasoning, Problem 2 required students to perform *unit-length iteration*—repeatedly counting consistent units of length along the sides of rectangular fences. This type of reasoning is foundational in developing accurate measurement skills for length.

These findings suggest that most students are still in the early stages of developing measurement reasoning. This highlights the need for structured, concrete, and conceptually grounded learning experiences. Lehmann (2023) emphasizes that advancing from intuitive to formal reasoning in area measurement requires scaffolded learning trajectories that progressively link students' visual, spatial, and numerical representations.

From an instructional perspective, this implies that students require more opportunities to work with physical manipulatives, simple measurement tools (e.g., rulers, unit blocks), and engage in guided discussions. Teachers should encourage students not only to find the correct answer but also to articulate their reasoning and strategies. In the context of composite shapes, a solid understanding of length iteration is essential for developing further skills, such as partitioning irregular shapes and calculating their area and perimeter.

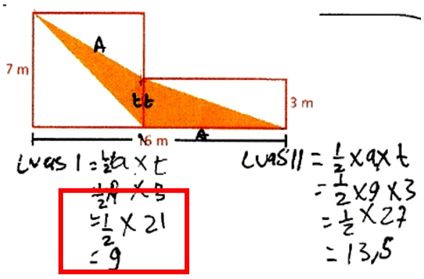
Thus, Activity 2 served not only to introduce procedures for measurement but also to support students' conceptual development toward more advanced mathematical reasoning. This aligns with the principles of the *Hypothetical Learning Trajectory* (HLT), in which learning activities are carefully sequenced to guide students progressively from informal understandings toward formal mathematical concepts through meaningful and contextualized experiences.

Post-Test

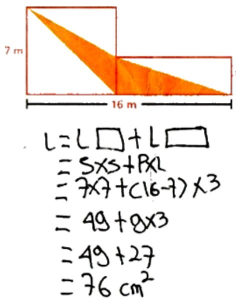
Following the implementation of the teaching experiment, the researcher administered a post-test to all participating students. Each student completed the test individually. The purpose of this assessment was to evaluate the extent to which students understood the area of composite shapes, as developed through the sequence of learning activities conducted in the two previous tasks. The post-test consisted of two main problems.

In Problem 1, students were asked to calculate the area of a composite figure based on the given diagram, with the question: *"Determine the area of the two polygons below."*

The visual structure of the diagram misled five out of the six students in this pilot experiment. They partitioned the figure into two shapes: a rectangle and a square. However, Azmal demonstrated a different understanding. He correctly interpreted the intended shapes as two orange-colored triangles. Figure 13 below shows students' responses to Problem 1 on the post-test.



(a) Azmal's



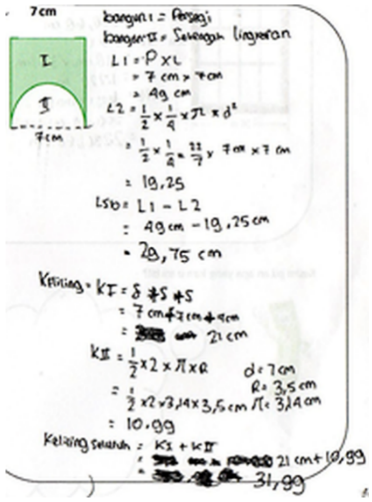
(b) Alief's

Figure 13. Students' responses to Post-Test Problem 1 (area of composite polygons)

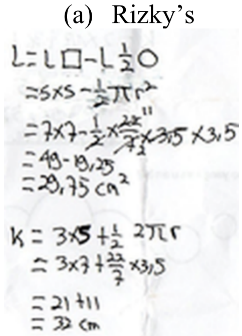
Azmal provided a nearly correct response. His error was minor and occurred in the final calculation of Area 1 (see the red box in Figure 13(a)). In contrast, Figure 13(b) displays an incorrect solution submitted by Alief. Since only Azmal was able to produce an accurate answer, it can be concluded that most students still struggled to correctly identify the composite shapes that were the focus of the problem.

The second problem in the post-test required students not only to calculate the area but also the perimeter of the two given polygons. The question stated: *"For these two polygons, in addition to calculating the area, determine their perimeter. Explain how you calculated it."*

In this problem, both Azmal and Alief successfully solved the task. They demonstrated a solid understanding of the concepts of area and perimeter and were able to explain their calculation strategies clearly. See Figure 14 for their responses.



(a) Rizky's



(b) Alief's

Figure 14. Students' responses to Post-Test Problem 2 (area and perimeter with explanation of strategy)

The post-test consisted of two main problems. The results of this pilot experiment indicated that students experienced notable improvement in their understanding of geometric concepts, both in articulating ideas and solving problems related to the area and perimeter of composite shapes.

In Problem 1, five out of six students misinterpreted the target figure. They saw "a rectangle and a square", whereas the intended decomposition was "two adjacent triangles". This outcome suggests that most students were still operating at what Hiele (1986) describes as the *visual level*: classifying shapes by appearance rather than explicit properties.

One student (Azmal) correctly organized the triangular structure and provided a nearly correct area calculation, with only a minor arithmetic slip. This suggests that Azmal had progressed to the *descriptive (analysis) level*, in which students begin to recognize and use properties of shapes to solve problems. This aligns with findings from Lehmann (2023), who observed that many primary students struggle to coordinate visual representations and quantitative reasoning when decomposing composite figures for area calculation.

In Problem 2, performance improved. Several students (e..g, Rizky and Alief) successfully completed both area and perimeter and articulated their reasoning. Others partially succeeded (for example, miscalculating the semicircle contribution). Importantly, many students could now describe *how* they got their answers, not just *what* the answers were. This signals growth toward descriptive/analytic levels of reasoning Hiele (1986) and supports claims in Battista (2007) that explicit discussion of decomposition and recomposition fosters conceptual advances in geometry.

These findings highlight the significant role of *visualization and spatial manipulation* activities in helping students move from intuitive to more structured conceptual reasoning. Lehrer, Jenkins, & Osana (1998) emphasized that instruction using visual representations and shape partitioning helps children develop spatial understanding and improve area measurement skills. Likewise, Sari & Ng (2022) observed that concrete area-manipulation tasks encourage students to move beyond simple shape recognition toward analytical reasoning about geometric relations and measurement.

Most notably, these findings align with the theoretical model proposed by Clements & Sarama (2004), who asserted that students' understanding of geometry develops progressively from visual perception to spatial and conceptual reasoning. They highlight the importance of manipulative-based learning, visual representations, and progressively sequenced exploratory activities to foster such development.

In this context, the two instructional activities implemented during the pilot experiment provided students with meaningful learning experiences that supported their gradual construction of geometric understanding. These activities activated spatial internalization processes—such as shape decomposition, length estimation, and measurement reasoning. This reinforces the argument by Clements & Sarama (2004) that geometric understanding and area measurement are not merely achieved through memorization of formulas, but through reflective and visual engagement with shapes and their structures.

IV. Conclusion

This study examined how primary students develop spatial and conceptual understanding of composite shapes through a structured instructional design. The focus extended beyond the correctness of students' answers to include their problem-solving strategies and reasoning processes, particularly how they visualized, decomposed, and restructured geometric figures.

The findings from both the teaching experiment and post-test indicated that most students transitioned from visual-based reasoning to more analytical strategies involving shape partitioning and unit iteration. This transition occurred primarily during Activity 2, where students transitioned from physically manipulating shapes (perceptual stage) to mentally decomposing composite figures and coordinating subregions (early spatial-structuring stage). These developments reflect the emergence of the four foundational area concepts—*partitioning, conservation, transitivity, and additivity*—as described in area learning trajectories. The post-test further confirmed this shift, indicating consolidation of analytic strategies, even when procedural accuracy had not fully developed. Such responses suggest genuine conceptual growth rather than purely procedural improvement.

By designing two sequential learning activities grounded in a Hypothetical Learning Trajectory (HLT), this research successfully bridges the gap between abstract theoretical constructs in geometry education and practical classroom implementation. The results show that well-structured visual and spatial activities can support students in constructing meaning, not merely memorizing formulas. Therefore, this study contributes to both the development of theoretical frameworks on early geometry learning and to practical insights for mathematics instruction that are cognitively responsive to how young learners think and reason.

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