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Analysis of the Level of Understanding of the Concept of the Isoline Method of Linear Program Material Based on APOS Theory in terms of Learning Interest

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Abstract

One of the methods that can be used to find solutions to linear programming models is the Graphical Method. Graphical methods are divided into the isoline method and the extreme point method. Based on initial observations and interviews, it was found that around 54% of students still needed help understanding the isoline method material in solving linear programs, especially when given real problems related to everyday life. In addition, students' learning interest in linear programming courses also varies. So the purpose of research in this article is to describe the level of understanding of students who have high, medium, and low learning interest in linear programming material, precisely the isoline method material, in determining the optimum solution. The method in this study is a qualitative method with a sampling technique using purposive sampling. Data collection was carried out using a questionnaire method for data on student learning interests and semi-structured test and interview methods to describe the level of student understanding based on APOS theory. The instruments in this study were questionnaires, written tests, and interview guidelines. Checking the validity of the data in this study used time triangulation. The collected data was analyzed using descriptive qualitative analysis. The results of the research show that: (1) subjects with high learning interest are at the level of schematic understanding; (2) subjects with interest in learning are at the level of understanding objects; and (3) subjects with low interest in learning are at the level of understanding of Action.

Keywords: isolin method; linear programming; APOS theory; interest in learning

I. Introduction

The Linear Programming method is the correct method for solving problems experienced by companies. The Linear Programming method helps companies by combining existing product variations based on the company's limited resources. Thus the company can carry out production optimally to obtain maximum profits (Cristian, 2013). The great benefits students get after taking this course positively impact the world of work, especially in the industrial sector. Based on these considerations, the linear programming course at Hasyim Asy'ari University is one of the compulsory subjects that must be taught by students of the mathematics education study program. Mathematics is a subject that emphasizes arithmetic, concepts, and problem-solving that can be used in everyday life (Dinullah & Ernawati, 2021). Mathematics is a subject that students must learn at every level of education (Amelia et al, 2021; Windasari et al., 2020). Mathematics is essential in various aspects of life (Kadafuk et al, 2020). Linear programming is a part of mathematics. The linear programming

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model can determine the value of the decision variables contained in the linear programming model. The methods used to find solutions from the linear programming model are divided into 2: the Graphical Method and the Simplex Method. The graphical method is used if the number of decision variables in the linear programming model is two decision variables. Meanwhile, the simplex method is used if at least two decision variables exist in the linear programming model (Cristian, 2013). The Graph Method is further divided into two, namely the isoline method and the extreme point method. This research focuses on the isoline method in solving the optimal solution problem. Because based on initial observations and interviews, it was found that around 54% of students still had difficulties related to the isoline method material in completing linear programs, especially if given real problems related to everyday life. Even though all mathematics material in schools and tertiary institutions contains aspects of understanding concepts because the essential ability to learn mathematics is to understand concepts first. Understanding concepts is essential; mastering concepts helps students learn mathematics (Ilyas & Basir, 2016). Conceptual understanding consists of two words, namely understanding, and concept. Understanding is the ability to understand at a higher level than knowledge (Sunandar, 2016). In the Big Indonesian Dictionary, understanding is the ability of intelligence to perceive situations or actions. Understanding is one of the cognitive dimensions in Bloom's taxonomy (Winata et al, 2020). Meanwhile, the concept is one of the main components of mathematics and plays a vital role in mathematics. The function of concepts in mathematics is to help understand something. Mathematical concepts can be formed through new construction along with old concepts. Understanding concepts form the basis of students' abilities based on mastery of the material in Mathematics so that students can re-express what they already know in a language that is easy to understand (Rosida & Pujiastuti, 2020 ; Ginting & Sutirna, 2021). Sumliyah & Rhosliana (2018) added that understanding mathematical concepts requires high generalization and abstraction skills. In addition, the level of interest in learning about linear programming courses also impacts students' understanding of the concept of the isoline method. Interest in learning is an encouragement from within students psychologically to learn something with full awareness (Lestari & Yudhanegara, 2017). So it must be classified as related to high, low, and moderate student interest in learning. High interest in learning will also have a high influence on student understanding. Likewise, students with moderate and low learning interests will affect student understanding. If students are interested in learning, it will be easier for them to understand the material (Komariyah, Afifah, & Resbiantoro, 2018). Student learning interest is a student's sense of interest in learning where the student wants to explore or do so that changes occur in the student (Sari & Harini, 2015)

In implementing the linear program material using the isoline method, it is necessary to have theoretical supporting factors in precise and careful learning to reinforce concepts to students. According to Mulyono (2012), a theory in learning is how to make students able to learn mathematical concepts well. One is the APOS Theory (Action, Process, Object, and Schema) (Mulyono, 2012; Silvia & Pranyata, 2021). In line with research conducted by Tall, which states that APOS theory contributes significantly to understanding mathematical cognition (Tall, 1999). Dubinsky et al (2005) also added that the main contribution obtained from the APOS analysis is an increased understanding of important aspects of human thought (Dubinsky et al, 2005). APOS theory has an essential place in the development of mathematics education in the context of its terms and phenomena.

For this reason, there have been many national and international studies based on the APOS Theory in the literature. While the theory has focused on international literature, studies on APOS Theory have been increasingly carried out

in national literature in the last ten years (Sefik et al, 2021; Arnawa et al, 2007). In addition, APOS theory has been widely used in analyzing the formation of mathematical concepts in universities. APOS theory discovers what goes on in students' minds when they are about to learn a mathematical concept, as well as the successes and failures that students encounter when completing mathematical tasks (Khairani, 2012; Syamsuri & Marethi, 2018; Umam & Susandi, 2022). Suppose two individuals have mastered a mathematical concept. With the APOS theory, it can be further detected who has a better mastery of the mathematical concept; that is, if someone can further explain the concept, then he is at a better level than the other (Handayani dkk, 2021). Furthermore, Lestari (2014) states that the goal to be achieved from the APOS theory is forming the learner's mental construction. This mental construction is the formation of an action, which is contemplated (interiorized) into a process, then encapsulated (encapsulated) into an object. The object can be decomposed again (de-encapsulated) into a process. Actions, processes, and objects can be organized into a schema, abbreviated as APOS.

APOS theory was founded by Dubinsky and developed by RUMEC (Research in Undergraduate Mathematics Education Community). According to this framework, learning occurs when an individual builds a mental structure through mental mechanisms. The most basic structure is Action, applied to mental objects constructed previously and directed by external stimuli. When an Action is repeated and reflected upon, it becomes a Process through the internalizing mental mechanism. Processes are, in turn, encapsulated into Objects so that Actions can be applied to them. A Schematic is a collection of coherent structures connected (Oktac et al, 2019). This theory was developed as part of an effort to understand how mathematics is learned. This theory also tries to understand how students construct different mathematical concepts and suggests pedagogic actions that stimulate learning (Salgado & Trigueros, 2015 ; Garcia & Parraguez, 2017). The presence of APOS theory begins with the hypothesis that mathematical knowledge is contained in individual tendencies related to situations of mathematical problems encountered by constructing actions, processes, and mental objects and organizing them in schemes to understand situations and solve these problems. These mental constructs are called the APOS Theory. This theory has been very useful in attempts to understand students' learning of various topics in calculus, abstract algebra, statistics, discrete mathematics, and other undergraduate mathematics fields (Dubinsky & McDonald, 2001). Parraguez and Octac (2010) add that this theoretical approach has been successfully used in research on learning mathematical concepts in Calculus, Analysis, Abstract Algebra, Discrete Mathematics, and Logic. APOS theory is interested in students' mental constructions when learning mathematical concepts (Parraguez & Oktac, 2010; Herawaty et al, 2020). APOS theory can also be used to analyze, from a cognitive perspective, classical problems related to the mathematical concept of infinity (Dubinsky et al., 2005). Arnon et al. (2014) added that the APOS Research Cycle consists of 3 stages, namely: (1) theoretical analysis, (2) instrument design and application, and (3) data collection and analysis.

Research that supports the APOS theory is conducted by Daud (2019), which concludes that the four components of the APOS Theory show a fairly positive perception score of Mathematics. These results imply that most students have an average attitude toward mathematics and can understand mathematics in their learning process (Daud et al, 2019). Another research that supports the APOS theory is conducted by Anwar et al (2018), which states that learning differential equations taught using the APOS theory approach is more effective than conventional methods (Anwar et al, 2013).

This study aimed to analyze students' conceptual understanding based on the APOS Stages in the isoline method material in

determining the optimum solution. Conceptual understanding becomes the basis for solving mathematical and contextual problems (Gusmania &Agustyaningrum, 2020). The level of understanding of the concept relates to a person's ability to comprehensively understand ideas or ideas, procedures, and facts that form a network of thinking with high interrelationships (Kuntarto, 2018). Through this research, lecturers will find exactly where the students' difficulties lie in understanding the concept of the isoline method in determining the optimum solution to a real problem.

II. Research Method

This research is descriptive qualitative research. A qualitative approach is a procedure in research where the data generated is written or oral words from the subject's observations. Meanwhile, descriptive research intends to investigate the circumstances, conditions, or other things mentioned; the results are presented as a research report. In this study, the presence of the researcher is significant because the researcher is the one who immediately acts and collects the data obtained. The researcher becomes the key instrument and acts as a non-participant observer, but the researcher does not involve himself directly in the life of the research object.

The implementation of this research took place at Hasyim Asy'ari University, Tebuireng, Jombang. The subjects of this study were 3 students from the class of 2019 who taught linear programming courses. Each had criteria for students with low, medium, and high interest in linear programming courses. Data collection consists of a questionnaire to determine student interest in learning, written tests, interviews, and documentation. The test is carried out by giving questions in the form of descriptions related to the isoline method material. Grouping at the level of understanding of the concept is divided into 3 categories: students with high interest, students with moderate interest, and students with low interest. For research subjects who will be interviewed, 3 students will be selected to get complete information on subject grouping. The determination of the categories of student interest in learning is presented in the following table:

Table 1.
Determination of student learning interest categories

Category	Provision
High	$X \geq \bar{X} + s$
Medium	$\bar{X} - s < X < \bar{X} + s$
Low	$X \leq \bar{X} - s$

Furthermore, indicators of understanding based on APOS theory adapted to the isoline method material in linear programming courses can be seen in table 2.

Table 2.
Isoline method concept construction indicators and extreme points in solving linear programming problems based on dubinsky's theory

APOS STAGE	INDICATOR
Action	• Students can correctly identify the questions they are working on • Students can understand the main questions in the problem.
Process	• Students can make plans to solve problems. • Students can exemplify variables based on problems. • Students can make mathematical formulations based on problems.
Object	• Students can work on problem-solving plans • Students can determine the method or strategy used.
Scheme	• Students can link actions, processes, and objects according to questions or problems. • Students can determine the correct answer and, according to the question • Students can provide conclusions from the steps taken in solving the problem.

The test questions amount to one description question that has been prepared based

on the ability to understand the concept and tested for validity by the validator. This test aims to collect information about the conceptual understanding of class 2019 students regarding the isoline method material in linear programming courses. The following are the test questions given to students.

An LSM appointed by TV OK surveyed the village and the city. There were at least 500 questionnaires distributed in villages. Questionnaires distributed in cities were equal to those distributed in villages. The number of questionnaires distributed was not more than 1500. Each respondent was paid 4000 for the village and 5000 for the city.

- *If TV is OK with paying LSM 6000 for each respondent, what is the minimum cost for LSM?*
- *What are the LSM's profits?*

Meanwhile, the interviews in this study were conducted to strengthen information and the results of students' work as information reinforcement when analyzing data. The type of interview used is the unstructured interview. This type of interview is an interview that is carried out freely with the arrangement of questions, and the words on each question can be changed according to the needs and conditions when the interview is carried out (Sugiyono, 2016). Then, documentation is used to support and complement the data collected.

Data analysis stages included reducing data, presenting data, drawing conclusions, and conducting the verification. Data reduction becomes a selection process in classifying students based on the types of errors made, simplifying and deleting unnecessary data, and finally, conclusions can be drawn and verified. Data presentation includes test results, the elaboration of sentences based on the research focus arranged in tables, and the recording of interview results. Drawing conclusions based on the data analysis provided through observation, test, interview, and data reduction results. Furthermore, checking the validity of the data is carried out using credibility criteria and through increased persistence and technical triangulation. This is important to do in order to obtain relevant data.

III. Results and Discussion

Before being given test questions and conducting unstructured interviews, as many as 13 students were given questionnaires to determine student interest in learning. Based on the data on the scores of interest in learning in linear program courses that have been collected, it is obtained = 72.84 and s = 15.54 so that the determination of the category of interest in learning in linear program courses is according to the following table:

Table 3.
Determination of student learning interest categories

Category	Provision
High	$X \geq 88{,}39$
Medium	$57{,}3 < X < 88{,}39$
Low	$X \leq 57{,}3$

After classifying, there were 2 students with high learning interest, 10 with moderate learning interest, and 1 with low learning interest. Then 3 students were selected from each category of interest in learning for further tests and structured interviews. The list of selected subjects is presented in Table 4.

Table 4.
Determination of student learning interest categories

No.	Subject	Gender	Category
1	M1	Female	High
2	M2	Female	Medium
3	M3	Female	Low

Based on the results of the data analysis that has been carried out, it shows that the subject's understanding in determining the solution to solving a linear program using the isoline method is in four specific stages of the APOS theoretical framework, namely Action, process, object, and scheme. The achievement of the level of understanding of the subject based on the APOS theory in terms of student learning interest is presented in table 5.

Table 5.
Achievement of subject understanding level based on APOS theory in terms of student learning interest

Category	Subject	Understanding Level			
		A	P	O	S
High	M1	√	√	√	√
Medium	M2	√	√	-	-
Low	M3	√	-	-	-

The following is a review of the subject's level of understanding based on APOS theory in each category of student interest in learning.

1. Level of Understanding of Subjects with High Learning Interest Based on APOS Theory

Subjects with high learning interest are subject M1. The subject has been able to make a mathematical model of problems related to linear programming, then find solutions using rules or formulas that need to be involved in finding solutions. The subject has understood how to determine the slope of the objective function line by choosing 2 specific points in the feasible region that satisfy $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ is then changed to $c_1x_1 + c_2x_2 + \ldots + c_nx_n = k$. Then draw the objective function line that hits that point. You can see picture 1 of the results of the answers from subject M1.

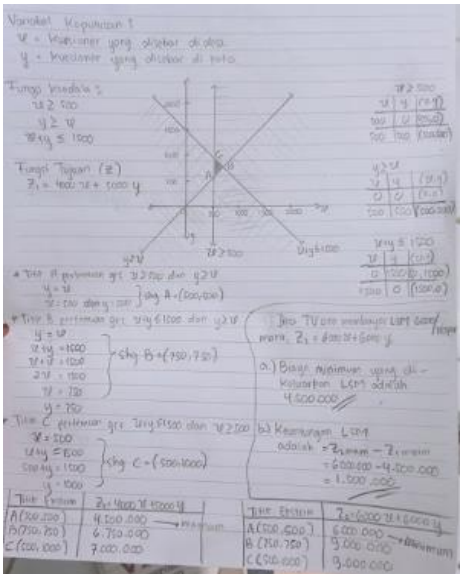

Figure 1. Results of student answers subject M1

Subject M1 can also determine the direction of increase/decrease of the objective function by selecting 2 lines (isolines) of the objective function in the feasible area and evaluating the values of the objective function on the two isolines. Furthermore, the subject can also follow the direction of increasing/decreasing until it reaches the boundary point (corner), where the increase/decrease of the objective function exits the feasible area. The subject has been able to find the optimum solution obtained from the boundary point where the increase/decrease of the objective function will leave the feasible area. Subject M1 can also choose the appropriate solution according to the given problem so that the solution can be found for this problem. M1 subjects can understand everything known in the problem well, so they can use it to find solutions to the given problems. Subject M1 can relate the relationship between the concepts of equations and inequalities, determine intersection points, and make graphs and algebraic concepts to solve these problems. In addition, Subject M1 is also able to use other strategies besides the isoline method, namely the extreme point method. Subject M1 can also conclude on the questions given. Subjects can organize actions, processes, and objects that they already have to solve problems related to the linear program so that, based on the APOS theory, subjects with high learning interests are at the level of schematic understanding. The results of semi-structured interviews reinforce this presentation to describe student understanding based on the APOS theory.

Researcher	:	*Is your final answer following what is required of the question?*
M1	:	*I am sure I have because I used two methods and the results were the same*
Researcher	:	*The last question is, is there a relationship between the isoline method and the corner point method?*
M1	:	*There is ma'am*
Researcher	:	*Are you sure?"*
M1	:	*Sure ma'am*

Researcher	:	*What is the relationship between the isoline method and the corner points?*
M1	:	*Both are graphical methods.*

Based on the triangulation process from the results of the work and the results of interviews with M1, it was found that M1 could answer the questions correctly. After being interviewed, M1 could connect the isoline method with the corner point method. As well as being able to describe that the two methods have a relationship because the two methods use the same graphical method.

2. Level of Understanding of Subjects with Medium Learning Interest Based on APOS Theory

Subjects with medium learning interest are M2 subjects. This M2 subject has also been able to make mathematical models of problems related to linear programming, then find solutions using rules or formulas that need to be involved in finding solutions. The subject also understands how to determine the slope of the objective function line by choosing 2 certain points in the feasible region that satisfy $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ is then changed to $c_1x_1 + c_2x_2 + \ldots + c_nx_n = k$. Then draw the objective function line that hits that point. You can see picture 2 of the results of the answers from subject M2.

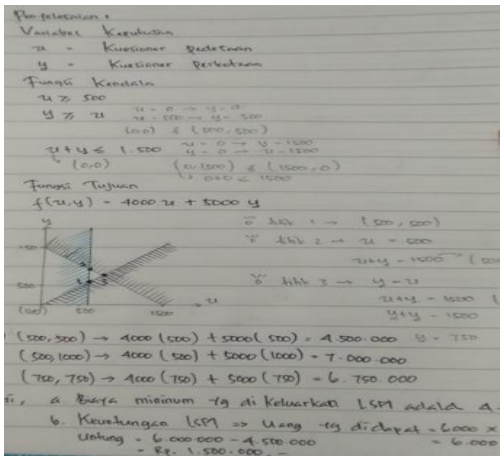

Figure 2. Results of student answers subject M2

Subject M2 can also determine the direction of increase/decrease of the objective function by selecting 2 lines (isolines) of the objective function in the feasible region and evaluating the values of the objective function on the two isolines. The subject can also follow the direction of increasing/decreasing until it reaches the boundary point (corner), where the increase/decrease of the objective function exits the feasible area. The subject has also found the optimal solution precisely obtained from the boundary point of the objective function. Internally, subject M2 already knows the steps/process for solving it. However, Subject M2 has not been able to make connections and has not been able to view the concept as an object. Subject M2 has not been able to organize the actions, processes, and objects that they already have to solve problems related to the linear program. Based on the APOS theory, subjects interested in learning are at the level of process understanding. The results of semi-structured interviews reinforce this presentation to describe student understanding based on the APOS theory.

Researcher	:	*Are you able to determine the results or answers that match the question*
M2	:	*I can ma'am*
Researcher	:	*Apart from the isoline method, can you use the extreme point method?*
M2	:	*yes ma'am*
Researcher	:	*OK, what conclusions can you get from this problem?*
M2	:	*In conclusion, madam, the minimum cost that the LSM must spend is four million five hundred thousand, and the profit earned is one million five hundred thousand.*
Researcher	:	*Is your final answer following what is required of the question?*
M2	:	*Sure ma'am*
Researcher	:	*Is there a relationship between the isoline method and the corner point method?*

M2 : *There is no ma'am.*
Researcher : *Are you sure?*
M2 : *Sure ma'am*

Based on the triangulation process of the results of the work and the results of interviews with M2, it was found that M2 could answer the questions correctly. However, after being interviewed, M2 could still not connect the isoline method with the corner point method. As well as not being able to describe that the two methods have a relationship because the two methods use the same graphical method. Even though M2 could complete it correctly, M2 did not understand conceptually but understood procedurally.

3. Level of Understanding of Subjects with Low Learning Interest Based on APOS Theory

Subjects with low interest in learning are subject M3. The subject has solved the given linear programming questions using the isoline and extreme point methods. However, the M3 subject has not been able to solve the problem with the isoline method correctly and correctly. This can be seen from the graphic form on the test sheet. The subject's activity in finding a solution is only limited to procedural activities. Therefore, based on the APOS theory, subjects with low learning interest are at the level of understanding of Action. This was also reinforced by the results of semi-structured interviews, which concluded that M3 subjects worked on the questions according to the procedures given by the lecturer. Internally, subject M3 is still unsure about the steps/process of completion that has been done.

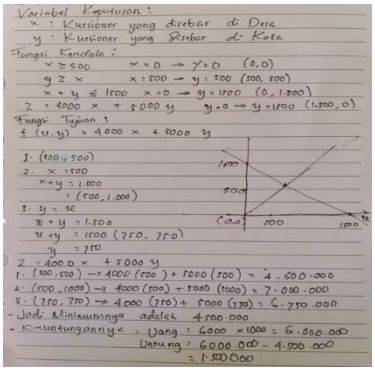

Figure 2. Results of student answers subject M3

The results of this study are supported by research conducted by Natalia (2017), which also concludes that subjects with high learning interest are at the schematic level of understanding; (2) subjects with interest in learning are at the process understanding level; and (3) subjects with low interest in learning are at the level of understanding of Action.

Researcher : *Can you determine the results or answers that match the question?"*
M3 : *yes ma'am*
Researcher : *Can you determine the results or answers that match the question?*
M3 : *I can, ma'am, but I only work with the corner point method*
Researcher : *Apart from the corner point method, can you use the isolation method*
M3 : *I can not, ma'am*
Researcher : *Lastly, is there a relationship between the isoline method and the corner point method?*
M3 : *No ma'am*
Researcher : *Are you sure?*
M3 : *Sure ma'am*

Based on the triangulation process of the work results and the results of interviews with M3, it was found that M3 could not answer questions using the isoline method. M3 can also still not connect the isoline method with the corner point method. As well as not being able to describe that the two methods have a relationship because the two methods use the same graphical method. The graphs that are made need to be corrected. So M3 is still in the Action category at the APOS stage.

From the study results, it can be explained that students with high learning interest in linear programming courses are at the schematic stage based on APOS theory, while students with moderate learning interest are at the moderate stage, and students with low learning interest are at the action stage. The results of this study are supported by research conducted by Natalia,

Sujatmiko, & Chrisnawati (2017), which also provides almost the same conclusions. The results of his research show that subjects with high learning interest in the quadratic equation material are at the level of schematic understanding; subjects with interest in learning are at the level of understanding the process; and subjects with low interest in learning are at the level of understanding of Action.

IV. Conclusion

Based on the analysis and discussion, the following conclusions are obtained: 1. The level of understanding of subjects with high learning interest based on APOS theory is at the schematic understanding level, 2. The level of understanding of subjects with moderate learning interest based on APOS theory is at the process understanding level, 3 The level of understanding of subjects with low learning interest based on APOS theory is at the level of understanding of Action

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