Jurnal Gantang IX (1) (2024): 83-94

JURNAL GANTANG

e-ISSN: 2548-5547
p-ISSN: 2503-0671

http://ojs.umrah.ac.id/index.php/gantang/index

Exploring students' understanding of Pascal's Mystic Hexagon Theorem within projective geometry

Amelia Defrianti Putri[1], Dadang Juandi[1*], Turmudi[1], Hazim Saputra Filia Reza[2]

[1] Universitas Pendidikan Indonesia, Bandung, Jawa Barat, 40154, Indonesia
[2] Universitas Negeri Padang, Padang, Sumatera Barat, 25171, Indonesia
*Corresponding Author: dadang.juandi@upi.edu

Submission: October 1st, 2024; Accepted: October 24th, 2024; Published: October 31st, 2024
DOI: https://doi.org/10.31629/jg.v9i1.6906

Abstract

This study investigates Blaise Pascal's contribution to mathematics, focusing on the Mystic Hexagon Theorem, and evaluates the understanding of Mathematics Education students at Padang State University (Class of 2021). A qualitative approach was employed, combining a systematic literature review and semi-structured interviews with 10 students selected from five classes. Student responses were analyzed and categorized into four levels: Expert, Medium, Low, and Unknowing. The findings reveal that only two students could successfully prove Pascal's Mystic Hexagon Theorem with clear and logical steps. At the same time, the remaining participants struggled to identify key concepts or complete the solution process. This outcome indicates students' significant lack of understanding of historical mathematical theories, highlighting an educational gap in integrating historical contexts into modern curricula. The study implies that incorporating historical mathematical theories, such as Pascal's contributions, into classroom instruction can improve students' problem-solving abilities, critical thinking, and geometric reasoning. Additionally, exposure to mathematical history fosters deeper appreciation and motivation, connecting abstract concepts to their historical evolution. Bridging this gap through history-based learning strategies can make mathematics more engaging and meaningful, enabling students to develop more substantial foundational knowledge and practical skills.

Keywords: Mystic Hexagon Theorem; projective geometry; Blaise Pascal

How to cite: Putri, A, D., Juandi, D., Turmudi., & Reza, H, S, F. Exploring students' understanding of Pascal's Mystic Hexagon Theorem within projective geometry. *Jurnal gantang, 9*(1), 83–94. https://doi.org/10.31629/jg.v9i1.6906

I. Introduction

The Babylonians were the pioneers of geometry studies (Kusaeri, 2017). Then followed by the Egyptians and the Ancient Greeks. These three nations are the pioneers of thoughts about geometry (Daulay et al., 2021). Some mathematicians we know in geometry pursue projective geometry, including Desargues and Pascal. Blaise Pascal is a mathematician with extraordinary abilities; he was born in Auvergne, France, in 1623 (Hodgkin, 2005; Saputro et al., 2015). He already showed his ability to observe phenomena surrounding mathematics at a young age, such as his investigation of some elementary geometry theorems at 12. He became a French Mathematical Society member (the French

Academy's forerunner) in 1666 at 14. At 16, he discovered several new theories in projective geometry problems, namely theorems related to curves (Hofmann et al., 1957).

Two years later, Pascal invented the first calculating machine, and since then, Pascal has contributed to the application of mathematics in the fields of mechanics and physics every year. In 1648, he wrote a problem about cone slices (Kusaeri, 2017). Pascal found this paper when he saw Desargues's work, but now it is gone. Then, in 1650, his health began to decline, and he decided to abandon his research and study religion more deeply. In the end, he returned to writing about mathematics, namely the arithmetic triangle (Traitedu Triangle Aritmetiqque), where this writing helped Fermat start the basics of mathematical probability theory and experiments with pressure problems (Asmara, 2019; Hodgkin, 2005).

Several mathematicians put their thoughts on projective geometry; one is Blaise Pascal (Putri, Yerizon, Arnellis, et al., 2024). The pioneer of projective geometry is Desargues, where, at that time, projective geometry was considered a "strange" science. Mathematics is a science that is closely related to human life because mathematics discusses facts, relationships, shapes, and spaces (Abdussakir et al., 2011; Prabowo, 2016). Jiang said that learning geometry is one of the areas of mathematics that is difficult to learn, so the author is interested in writing this paper in the hope that it can facilitate readers in understanding geometry material (Fardian & Dasari, 2023; Nur'aini et al., 2017)

The purpose of this research is to make students care about the origin of a mathematical theorem and motivate readers to study history, especially in the field of mathematics (Asngari, 2015; Efendi et al., 2021; Wahyu & Mahfudy, 2016). Seeing the development of geometry today, researchers are interested in taking the topic of collinear points on hexagons on a circle (Kusaeri, 2017). This topic is rarely discussed in previous in-depth studies because Pascal is better known for the binomial theorem and Pascal's triangle (Zainal et al., 2021). Pascal's Mystic Hexagon Theorem is rarely discussed, but without realizing the use of this theorem, it is sometimes used when working on cone wedge problems. This study is a form of researcher's interest in one of Pascal's discoveries, namely proving the existence of the colinearity of three points that intersect the hexagon in a circle. In addition, the researcher wants to know the extent of today's students' understanding of the application of Pascal's mystical hexagon theorem. As we know, technology is developing rapidly (Amboro, 2020; Fardian, Herman, et al., 2024; Putri, Juandi, Jupri, et al., 2024); this makes some people ignorant of the historical development of mathematics (Efendi et al., 2021; Firdaus, 2021; Hidayati & Irawanto, 2008).

II. Research Method

This paper's research type is qualitative research with a literature review and case study methods. Qualitative research is conducted to collect non-numerical qualitative data (Somantri, 2005). Researchers used several methods in this study: (a) collecting information using the literature study method or Systematic Literature Review (SLR) from various books and journals regarding Pascal's biography and discoveries. (b) Conducting interviews with ten respondents (from five classes in the Mathematics Education study program at Padang State University Class of 2021, two people from each class were selected) regarding their knowledge of problem-solving for Pascal's Theorem, namely the Mystic Hexagon.

This study aims to determine the extent of student knowledge about solving or proving Pascal's mystical hexagon theorem. The instrument used is an open interview, which is conducted to obtain accurate data from the appropriate source. Data analysis in this study was carried out by exploring in-depth information about questions related to the main topic, parsing data into units, categorizing them, choosing which ones to study, and drawing conclusions that can be presented to others. The data analysis process begins with the researcher asking questions and then giving these questions to respondents (two students from each class in the Mathematics Education Study Program at Padang State University, Class of 2021).

III. Results and Discussion

Blaise Pascal's Discoveries

Blaise Pascal is the first great prose classic in modern French, Lettres Provinciales and Pens'ees. For most of his life, his research was hampered by his illness. His mathematical popularity is more about what he did than what he achieved. Pascal's mother died when he was three years old, and he was raised by his father, Etienne Pascal, a judge and mathematician. Pascal grew up without attending school or studying at university; his father taught him exclusively at home (Saputro et al., 2015). Pascal invented a device called a calculating machine in 1642; this machine was created by Pascal to help his father work in calculating government money in Rouen; at that time, this machine was only able to calculate numbers that did not exceed six digits (Fardian, Suryadi, et al., 2024; Zainal et al., 2021).

The "Mystic Hexagon" of projective geometry is one of the great discoveries made by Pascal, whose theorem reads: "If a hexagon is formed in a cone, then the points of intersection of the three pairs of faces will lie on a line (collinear), and vice versa ."The theorem has derived more than 400 theorems resulting from projective geometry. This paper was never published because he considered that his paper would never be complete (Hodgkin, 2005; Putri, Juandi, & Turmudi, 2024b).

Later, in 1653, Pascal wrote "Traite du Triangle Arithmeticus," this paper was printed in 1665. Pascal arranged triangles arithmetically like a number chart; an element in the second row or the next row is obtained from the sum of all the elements above it, for example, 35 = 15 + 10 + 6 + 3 + 1 (element in the fourth row of the fifth column located in the number arrangement chart), 126 = 56 + 35 + 20 + 10 + 4 + 1 (element in the sixth row of the fifth column). Triangular figures can be formed by drawing a diagonal line, which algebraically tells us that the numbers along the diagonal are the binomial coefficients (Hodgkin, 2005). Examples of the numbers along the third, fourth, and fifth diagonals are as follows:

$$(a+b)^2 = a^2 + 2 + b^2, \quad (1,2,1)$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3, (1,3,3,1)$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^2 + b^4, \quad (1,4,6,4,1)$$

The binomial coefficients obtained are one of the applications of Pascal's triangle (arithmetic triangle). Pascal's triangle can be used to determine probabilities, for example, to obtain the magnitude of the combination of n against r (the number of combinations of n different objects with one take of r objects). Where the statement can be expressed in the form:

$$\frac{n!}{r!\,(n-r)!}$$

With n! is the multiplication: $n\,.\,(n-1)\ .\,(n-2)\ .\,(n-3)\ldots(3)\ .\,(2)\ .\,(1)$

We must know that Pascal was not the first person to discuss the arithmetic triangle, but Chu Shi-Kie (1303), a Chinese algebraist. Based on that old reference, Pascal developed the properties and applications of arithmetic triangles as "Pascal's Triangle" (Hodgkin, 2005). Another discovery made by Pascal is the "Cycloid," a curve obtained from shifting a point around a circle that rotates along a straight line. Mathematically, this curve plays an important role in calculus calculation methods (Putri, Juandi, & Turmudi, 2024a).

At 16, Pascal tested his mathematical skills before the circle of Augustus Mersenne by offering a discussion on a leaflet entitled Essai Pour Les Coniques. The leaflet was printed in 1640 on a single sheet of paper, one of the most useful pages in the history of mathematics. The essay contains statements of a number of general theorems of a projective nature, one of which is Pascal's mystical hexagon theorem (Hofmann et al., 1957). The author will discuss one of Pascal's discoveries more deeply, namely the Mystic Hexagon or Pascal's Theorem. This theorem discusses the linearity of a line on a cone slice by selecting six points on the cone slice. This theorem also sees the existence of the collinearity of the intersection of six points on the circle (Hofmann et al., 1957).

It is said that if a hexagon is in a circle, the three points of intersection of pairs of intersecting (non-parallel) sides are on a straight line. Figure 1 below shows a hexagon on a circle that yields the mystical hexagon theorem.

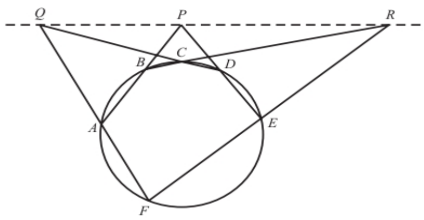

Figure 1. Pascal's Mystic Hexagon Theorem

Pascal closed his essay by saying, "There are many other problems and theorems, many deductions that can be made from what has been stated above" (Nurjanah et al., 2021). Based on Pascal's statement, one of the other deductions from the mystic hexagon problem is as follows:

Let A, B, C, D, E, and F be six points on a circle (the points may not be consecutive). Suppose $P = AB \cap DE$, $Q = BC \cap EF$, *and* $R = CD \cap FA$. *Then, the points P, Q, and R are collinear. Notes:* $AB \cap CD$*=intersection of AB with CD*

Proof:

Consider Figure 2, which modifies Pascal's mystic hexagon theorem.

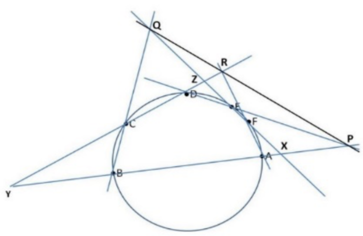

Figure 2. Another Form of Proof of Pascal's Mystical Hexagon Theorem

Suppose X=EF∩AB, Y=AB∩CD, and Z=CD∩EF, view ΔXYZ with BC as the transverse axis, then obtain:

$$\frac{ZQ}{QX} \cdot \frac{XB}{BY} \cdot \frac{YC}{CZ} = -1$$

Similarly, ΔXYZ with the transversal lines FA and DE yields a similar equation. Then, when we multiply the three equations, we get:

$$\frac{ZQ}{QX} \cdot \frac{XB}{BY} \cdot \frac{YC}{CZ} \cdot \frac{XP}{PY} \cdot \frac{YD}{DZ} \cdot \frac{ZE}{EX} \cdot \frac{YR}{RZ} \cdot \frac{ZF}{FX} \cdot \frac{XA}{AY} = \frac{ZQ}{QY} \cdot \frac{XP}{PY} \cdot \frac{YR}{RZ} = -1$$

Since $XA.XB = XE.XF$, $YC.YD = YA.YA$, and $ZE.ZF = ZC$ Based on Menelaus' theorem, ZD proves that P, Q, and R are collinear. This is one way to prove Pascal's theorem; there are many ways to find evidence that points P, Q, and R are collinear (Nurjanah et al., 2021).

Category Of Students' Knowledge Of Pascal's Mystic Hexagon Theorem

This section will show ten answers from Mathematics Education students, State University of Padang Batch 2021, randomly selected with details of two people from each class. Previously, the researcher used several codes to write the results of the interviews that had been conducted, namely P for researchers and a combination of letters and numbers for students; for example, A1 means student 1 from class A, and so on.

Student 1 from class A

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

A1: *Not yet*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

The answer of student 1 from class A is shown in Figure 3

Figure 3. Representation of student 1's answer from class a to Pascal's Mystic Hexagon Theorem

Based on the answers shown, the student could identify the problem, but he had difficulty taking the next step, so he could not solve the given problem.

Student 2 from class A

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

A2: *Never been*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

The answer of student 2 from class A is shown in Figure 4

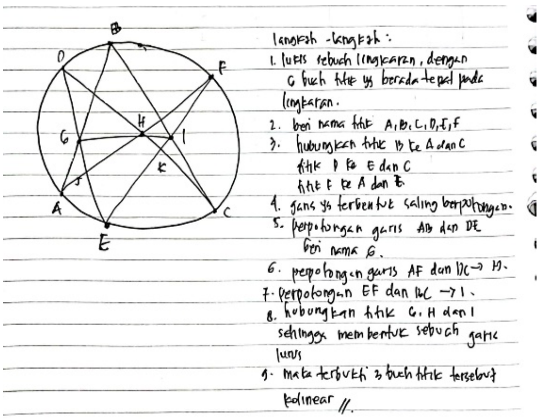

Figure 4. Representation of student 2's answer from class a to Pascal's Mystic Hexagon Theorem

Based on the answer shown by student 2 from class A, it is sufficient to prove the linearity of three intersecting points. The right steps accompany the answer shown. Thus, it can be concluded that he can prove the given theorem.

Student 3 from class B

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

B3: *Never been*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

The answer of student 3 from class B is shown in Figure 5

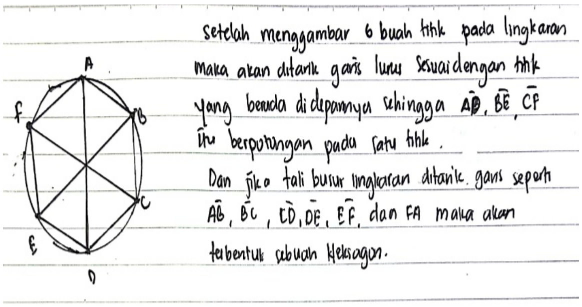

Figure 5. Representation of student 3's answer from class B to Pascal's Mystic Hexagon Theorem

Based on the answers, students only form a hexagon inside a circle but do not see the next step. So, student 3 from class B cannot prove the given theorem.

Student 4 from class B

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

B4: *Not yet*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

The answer of student 4 from class B is shown in Figure 6

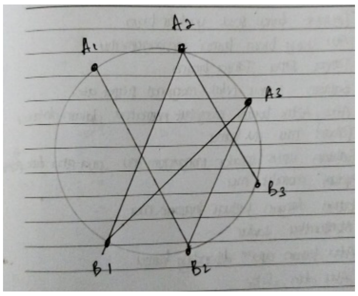

Figure 6. Representation of Student 4's Answer from class B to Pascal's Mystic Hexagon Theorem

Based on the answer, the student arrived at the process of connecting the points on the circle's circumference; this almost leads to the final step of the proof (Scristia et al., 2023). However, it can be concluded that the student has not been able to prove the theorem given.

JURNAL GANTANG. October 2024; IX(1): 83 – 94
p-ISSN. 2503-0671
e-ISSN. 2548-5547

Student 5 from class C

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

C5: *Never been*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

The answer of student 5 from class C is shown in Figure 7

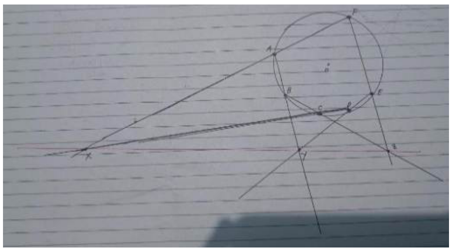

Figure 7. Representation of student answer 5 from class C to Pascal's Mystic Hexagon Theorem

Based on the answers that have been shown, the student only forms a hexagon inside a circle and then connects the points with several lines, but the next step has not been seen. So, student 5 from class C could not prove the given theorem.

Student 6 from class C

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

C6: *Never been*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

The answer of student 6 from class C is shown in Figure 8

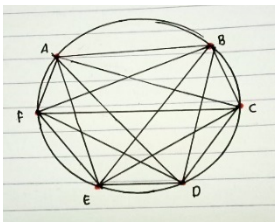

Figure 8. Representation of Student Answer 6 from class C to Pascal's Mystic Hexagon Theorem

Based on the answers shown, the student only formed a hexagon inside a circle and then connected the points with several lines, but what was seen was four intersecting points, which should have been three points. So, student 6 from class C cannot prove the given theorem.

Student 7 from class D

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

D7: *Never been*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

The answer of student 7 from class D is shown in Figure 9

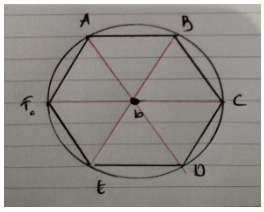

Figure 9. Representation of student answer 7 from class D to Pascal's Mystic Hexagon Theorem

Based on the answer shown by student 7 from class D, it is enough to prove the collinearity of three intersecting points, where he extends the line outside the circle, and three points are on the same line (collinear). So, it can be concluded that he can prove the given theorem.

Student 8 from class D

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

D8: *Never been*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

The answer of student 8 from class D is shown in Figure 10

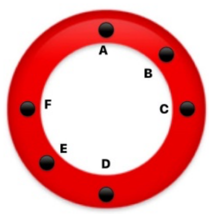

Figure 10. Representation of student answer 8 from class D to Pascal's Mystic Hexagon Theorem

Based on the answer shown, the student struggled to do the next step; he stopped placing the points on the circle's circumference only, so he could not solve the given problem.

Student 9 from class E

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

E9: *Never at all*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

E9: *I don't know what to do; I can't do this problem*

Student 10 from class E

P: *Have you heard or read about the "Mystic Hexagon Theorem"?*

E10: *Never at all*

P: *How do you prove the continuity of three points that intersect a hexagon inside a circle?*

E10: *I can't solve the problem; I don't understand the problem here.*

Students 9 and 10 from class E's answers showed they did not know what steps to take, so they could not solve the theorem problem.

From the interviews conducted by the researcher above, the researcher can compile several categories of answers from all respondents, where the researcher gets four categories, which are the results of the research findings. These categories are shown in Table 1

Table 1. Categorization of Students Based on Answers to the Problem of Pascal's Mystical Hexagon Theorem

No	Category	Description
1	*Expert in the Mystic Hexagon Pascal Theorem*	Students have never heard of the theorem but can prove Pascal's Mystic Hexagon Theorem correctly with clear steps
2	*Medium in the Mystic Hexagon Pascal Theorem*	Students have never heard of the theorem and have not been able to prove Pascal's Mystic Hexagon Theorem, but have carried out the problem identification process and reached the stage of making hexagons
3	*Low in the Mystic Hexagon Pascal Theorem*	Students have never heard of the theorem and have not been able to prove Pascal's Mystic Hexagon Theorem, but have done the problem identification process by making a circle and six points.
4	*Unknowing in the Mystic Hexagon Pascal Theorem*	Students have never heard of the theorem and cannot solve Pascal's mystic hexagon theorem or identify the problem.

The research results presented in the table above aim to be used as findings. Therefore, the results of this research are associated with relevant theories that previous researchers have carried out. The main focus of this research is to determine students' understanding of Pascal's Mystic Hexagon Theorem, which has been categorized into four categories, namely Expert in the Mystic Hexagon Pascal Theorem, Medium in the Mystic Hexagon Pascal Theorem, Low in the Mystic Hexagon Pascal Theorem, and Unknowing in the Mystic Hexagon Pascal Theorem. For this reason, researchers will discuss the findings and compare them with previous studies. The explanation of the categorization prepared by the researcher is as follows:

1. Expert in the Mystic Hexagon Pascal Theorem

The researcher's findings regarding Expert in the Mystic Hexagon Pascal Theorem where students do not know Pascal's mystic hexagon theorem but can prove the theorem. This category is supported by a journal that analyzes student difficulties in answering mathematics history problems titled "Analysis of Student Difficulties in Mathematics History Courses" (Warmi, 2018). The type of student in this category does not experience the three things that become obstacles for students in answering problems because they understand the concept, know how to use it, and can communicate it properly and correctly.

In this type, students can identify the problems and then utilize the knowledge learned before to prove the theorem with systematic steps. In accordance with the results of the interviews above, students who are included in this category are student 2 from class A and student 7 from class D.

Students in this category can show links between knowledge (mastery of geometry concepts), the use of correct geometry concepts, and communication systems in answering math problems, especially geometry (Wahyu & Mahfudy, 2016). Students in this category showed no difficulties, as shown in the journal "Analysis of Student Difficulties in History of Mathematics Course" (Warmi, 2018).

2. Medium in the Mystic Hexagon Pascal Theorem

The researcher's findings regarding Medium in the Mystic Hexagon Pascal Theorem are that students do not know Pascal's mystic hexagon theorem and cannot prove the theorem. However, the answers lead to the proof's final story, supported by a journal that analyzes student difficulties in answering mathematics history problems titled "Analysis of Student Difficulties in Mathematics History Courses" (Warmi, 2018). This type of student in this category experienced little constraints on the use of concepts and the ability to communicate. Students in this category showed confusion about how to communicate answers in mathematical form but already knew the initial concepts to solve the problems that had been given (Saraswati et al., 2020).

In this type, students can identify the problems given and then utilize the knowledge learned before but are constrained in the next step of solving (Afwan et al., 2020; Jufri & Dasari, 2023). In accordance with the interview results above, students included in this category are student 3 from class B, student 4 from class B, student 5 from class C, and student 6 from class C. Students in this category can show links between previously held knowledge and try to carry out preliminary analysis but are constrained in the final resolution of the problem (Efendi et al., 2021; Nur'aini et al., 2017; Wahyu & Mahfudy, 2016).

3. Low in the Mystic Hexagon Pascal Theorem

The researcher's findings regarding Low in the Mystic Hexagon Pascal Theorem are that students do not know Pascal's mystic hexagon theorem and cannot prove it. This research is supported by a journal that analyzes student difficulties in answering mathematics history problems, "Analysis of Student Difficulties in Mathematics History Subjects" (Warmi, 2018).

This type of student in this category has a little problem understanding the concept, so he does not know how to use the concept he has understood (Saraswati et al., 2020). As a result, students in this category are also constrained in carrying out the communication process (answering math problems correctly). In this type, students can identify the problems given and then utilize the knowledge they have learned but are constrained in the next step at the beginning of the proof process. According to the interview results above, students included in this category are students 1 from class A and 8 from class D.

Students in this category can show links between previously known knowledge. However, they doubt using concepts because they do not optimally understand the geometry concepts taught (Wahyu & Mahfudy, 2016). Thus, students in this category can be said to experience problems using geometry concepts, lack communication skills, and have few problems understanding geometry concepts (Efendi et al., 2021).

4. Unknowing in the Mystic Hexagon Pascal Theorem

The researcher's findings regarding Unknowing in the Mystic Hexagon Pascal Theorem are that students do not know Pascal's mystic hexagon theorem and cannot prove it. This research is supported by a journal that analyzes student difficulties in answering mathematics history problems, "Analysis of Student Difficulties in Mathematics History Courses" (Warmi, 2018).

The type of students in this category experience the three obstacles mentioned in the journal, where students do not know the concept. As a result, this impacts the use of concepts and the inability to communicate (Nofindra, 2019). That is, in this type, students have not been able to identify the problems given then have not fully utilized the knowledge that has been learned, so they are constrained at the beginning of the proof process (Putri, Juandi, Turmudi, et al., 2024). In accordance with the interview results above, students included in this category are student 9 from class E and student 10 from class E.

Students in this category have been unable to show links between their knowledge, application of concepts, and poor communication skills (Wahyu & Mahfudy, 2016). This can also be caused by low learning motivation in students, perhaps in terms of learning targets, learning attention, learning environment, and interest in the material being taught (Argaswari, 2018; Efendi et al., 2021; Fachrudin & Kusumawati, 2018; Saraswati et al., 2020).

IV. Conclusion

In conclusion, Pascal's Mystic Hexagon Theorem demonstrates the collinearity of three intersecting points on a hexagon inscribed in a circle, which can be approached through various methods depending on the understanding and perspective of the learner. This study found that students' understanding of the theorem varied significantly, categorized into four levels: Expert, Medium, Low, and Unknowing. The majority struggled with the application and proof, highlighting the need for better integrating historical mathematical concepts into the curriculum.

Studying the history of mathematics is not merely about learning past events; it fosters critical thinking, improves problem-solving skills, and develops a deeper appreciation for mathematics as a dynamic field. Incorporating mathematical history into classroom instruction can create a more engaging and reflective learning environment, enabling students to connect theoretical knowledge with practical applications. This approach enhances conceptual understanding and encourages a more holistic perspective on mathematics and its role in everyday life.

Acknowledgment

The authors wish to express their gratitude to the Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia with Program Magister Menuju Doktor untuk Sarjana Unggul (PMDSU) Batch VII and the Directorate of Research, Technology, and Community Service (DRTPM), which has provided funding support and opportunities.

Reference

Abdussakir, A., Aliviana, R., & Rohmawati, Y. (2011). *Analisis matematik terhadap azimat numerik dan alfabetik*.

Afwan, B., Suryani, N., & Ardianto, D. T. (2020). Analisis kebutuhan pembelajaran sejarah di era digital. *Proceeding Umsurabaya*.

Amboro, K. (2020). Sejarah Publik dan Pendidikan Sejarah Bagi Masyarakat. *Historis: Jurnal Kajian, Penelitian Dan Pengembangan Pendidikan Sejarah*, *5*(1), 20–28.

Argaswari, D. P. A. D. (2018). Integrasi sejarah matematika untuk meningkatkan atensi siswa. *Indonesian Journal of Mathematics Education*, *1*(1), 59–65.

Asmara, Y. (2019). Pembelajaran Sejarah Menjadi Bermakna dengan Pendekatan

Kontektual. *Kaganga: Jurnal Pendidikan Sejarah Dan Riset Sosial Humaniora*, *2*(2), 105–120.

Asngari, D. R. (2015). Penggunaan geogebra dalam pembelajaran geometri. *Seminar Nasional Matematika Dan Pendidikan Matematika UNY*, 299–302.

Daulay, L. A., Nasution, A. K. P., Tohir, M., Simamora, Y., & Saragih, R. M. B. (2021). Geogebra assisted blended learning on students' spatial geometry ability. In S. null (Ed.), *Journal of Physics: Conference Series* (Vol. 1839, Issue 1). IOP Publishing Ltd. https://doi.org/10.1088/1742-6596/1839/1/012009

Efendi, A., Fatimah, C., Parinata, D., & Ulfa, M. (2021). Pemahaman gen z terhadap sejarah matematika. *Jurnal Pendidikan Matematika Universitas Lampung*, *9*(2), 116–126.

Fachrudin, A. D., & Kusumawati, I. B. (2018). Pendekatan Geometris Yang Melibatkan Sejarah Matematika Dalam Pembelajaran Penyelesaian Persamaan Kuadrat. *Jurnal Pembelajaran Matematika*, *5*(1).

Fardian, D., & Dasari, D. (2023). The effects of problem-based learning on mathematical proficiency: A combined bibliometric analysis and meta-analysis review. *Journal of Didactic Studies*, *1*(2), 99–113.

Fardian, D., Herman, T., Suryadi, D., & Prabawanto, S. (2024). Gamifying Mathematics: A hermeneutic phenomenological study on Role-Playing Games in Linear Equations. *PYTHAGORAS Jurnal Pendidikan Matematika*, *19*(1).

Fardian, D., Suryadi, D., Prabawanto, S., & Hayuningrat, S. (2024). Research trends on early algebra in the middle school: A combined bibliometric and meta-analysis review. *Jurnal Elemen*, *10*(2), 410–440.

Firdaus, D. R. (2021). *Pentingnya Sejarah Bagi Generasi Muda*.

Hidayati, Y., & Irawanto, B. (2008). Geometri proyektif pg (2, pn) untuk membentuk rancangan blok tidak lengkap seimbang simetris. *jurnal matematika dan komputer*, *11*(3), 121–125.

Hodgkin, L. (2005). *A history of mathematics: from Mesopotamia to modernity*. OUP Oxford.

Hofmann, J. E., Gaynor, F., Midonick, H. O., & Beyer, R. T. (1957). The history of mathematics. *Physics Today*, *10*(9), 38.

Jufri, L. H., & Dasari, D. (2023). Evaluation of the implementation of machine learning algorithm K-Nearest Neighbors (KNN) using rapid miner on junior high school student learning outcomes. *Jurnal Gantang*, *8*(2), 193–197. https://doi.org/10.31629/jg.v8i2.6590

Kusaeri, K. (2017). *Historiografi matematika: Rujukan paling otoritatif tentang sejarah perkembangan matematika*. Matematika Yogyakarta.

Nofindra, R. (2019). Ingatan, lupa dan transfer dalam belajar dan pembelajaran. *Jurnal Pendidikan Rokania*, *4*(1), 21–34.

Nur'aini, I. L., Harahap, E., Badruzzaman, F. H., & Darmawan, D. (2017). Pembelajaran matematika geometri secara realistis dengan GeoGebra. *Matematika: Jurnal Teori Dan Terapan Matematika*, *16*(2).

Nurjanah, D., Nurjanah, E., Hasan, A. F., Nabila, A., & Ariany, R. L. (2021). Kontribusi sejarah aljabar Babilonia dan aljabar Arab terhadap berpikir aljabar. *Jurnal Analisa*, *7*(2), 112–123.

Prabowo, A. (2016). Menalar Sengkala Merajut Matematika. *Universitas Jendral Soedirman*.

Putri, A. D., Juandi, D., Jupri, A., & Muchsin, S. B. (2024). Mastering the TPACK framework: Innovative approaches by mathematics teachers. *Jurnal Elemen*, *10*(3), 582–594.

Putri, A. D., Juandi, D., & Turmudi, T. (2024a). Blended learning dalam pembelajaran matematika : a systematic literature network analysis. *7*(3), 501–516. https://doi.org/10.22460/jpmi.v7i3.21992

Putri, A. D., Juandi, D., & Turmudi, T. (2024b). Realistic mathematics education and mathematical literacy: a meta-analysis conducted on studies in Indonesia. *Journal of Education and Learning (EduLearn)*, *18*(4), 1468–1476.

Putri, A. D., Juandi, D., Turmudi, T., Fardian, D., & Rizki, L. M. (2024). Does The Realistic Mathematics Education Approach Enhance Students' Mathematical Literacy? A Systematic Literature Review. *Avalokana Journal of Literature Review*, *1*(1), 30–37.

Putri, A. D., Yerizon, Y., Arnellis, A., & Suherman, S. (2024). Development of realistic mathematics education-based teaching materials to increase students' mathematical literacy ability. *AIP Conference Proceedings*, *3024*(1).

Saputro, B. A., Prayito, M., & Nursyahidah, F. (2015). Media pembelajaran geometri menggunakan pendekatan pendidikan matematika realistik berbasis GeoGebra. *Kreano, Jurnal Matematika Kreatif-Inovatif*, *6*(1), 34–39.

Saraswati, R. R., Nurizzah, N., Pitnawati, P., & Habibah, U. (2020). Integrasi sejarah matematika dalam pembelajaran matematika pada materi phytagoras. *Risenologi*, *5*(1), 9–13.

Scristia, S., Dasari, D., & Herman, T. (2023). Sentiment analysis of prospective mathematics teacher on reasoning and proof questions using Naïve-Bayes classification. *Jurnal Gantang*, *8*(2), 155–165. https://doi.org/10.31629/jg.v8i2.6591

Somantri, G. R. (2005). Memahami metode kualitatif. *Makara Human Behavior Studies in Asia*, *9*(2), 57–65.

Wahyu, K., & Mahfudy, S. (2016). Sejarah matematika: Alternatif strategi pembelajaran matematika. *Beta: Jurnal Tadris Matematika*, *9*(1), 89–110.

Warmi, A. (2018). Analisis kesulitan mahasiswa pada mata kuliah sejarah matematika. *JP3M (Jurnal Penelitian Pendidikan Dan Pengajaran Matematika)*, *4*(1), 7–14.

Zainal, Z., Halik, A., Sultan, M. A., & Resa, A. (2021). Development of Geometry Flipbook Multimedia Learning Media for Elementary School Students. *Journal of Educational Science and Technology*, *7*(2), 207–215.

JURNAL GANTANG. October 2024; IX(1): 83 – 94
p-ISSN. 2503-0671
e-ISSN. 2548-5547