Analysis of Student Errors in Solving Basic Logarithmic Problems Using Kastolan Error Analysis

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Abstract
This research aims to describe the types of student errors in solving logarithmic problems. The subjects in this study were class X students of SMAN 15 Pekanbaru City, Riau Province. The research method uses a qualitative descriptive approach. Data collection techniques using written tests and documentation. Data analysis used descriptive qualitative analysis by compiling various kinds of student error data as material for research. In this analysis, the types of student errors are categorized based on Kastolan stages. Based on the results of the study, there are still many students who need help solving logarithmic problems. The most errors were conceptual errors at 74.14%, followed by procedural errors at 15.52%, and types of errors with the least technical errors at 10.34%. Conceptual errors indicate a weak understanding of the properties of logarithms, so they cannot apply them. The second type of procedural error occurs due to unsystematic settlement and exit from a predetermined procedure. The kind of error with the lowest percentage is a technique that shows the accuracy of students' calculations in solving problems. This happens because of a need for more thoroughness with the signs that must be used.

Keywords: error analysis; kastolan error analysis; logarithm

I. Introduction
The ability of a teacher to perceive and respond to student errors is essential and is always used in learning mathematics if misconceptions occur in a learning process. More than just knowing students' mistakes is required; they must be able to analyze and identify them. According to (Aziz., 2020; Kristofofa & Sujadi, 2017; Andriani, Suastika, & Sesanti, 2017; Wijaya, 2016; Lestari & Prahmana, 2018), students' mistakes in solving problems are caused by students' mistakes in translating a mathematical concept. Therefore, the speed of identifying the causes of student errors is needed, especially in applying mathematical concepts and principles.

One of the goals of learning mathematics that teachers consider essential is mastery of the mathematical concepts. Mastery of mathematical concepts is considered an essential ability to be understood in mathematics. Concept understanding aims to develop students' abilities in learning mathematics and solving mathematical problems by linking concepts to other concepts.

The logarithm is a branch of mathematics whose calculation process is the opposite of calculating the power (exponent). In addition, in general, in the process, logarithms are operated based on various properties and forms. Each calculation process requires understanding the concepts and principles that students must...
understand, such as the properties of logarithms. Not understanding the properties of logarithms is included in not understanding the concept. A logarithm problem must be solved with at least one property, but every logarithm problem-solving generally applies more than one property.

The problem of students’ understanding of logarithmic material in terms of errors in solving problems has been found in previous research reports. The results of the study stated that students made two errors; namely, students were unable to generalize because they needed to determine the method that should be used, and students needed to write down the final results (Gunawan & Fitra, 2021). Mistakes are often made because some students still need help understanding how to correctly apply the properties of logarithms (Hayati & Budiyono, 2018).

Based on the results of a literature review conducted by researchers, in the research of Supardi, Gusmania, & Amelia (2019) and Tonda, Suwanti, & Murniasih (2020), the results show that the achievement of student test results is meager, as evidenced by the test scores which are still below the KKM. After being traced, it is known that the most challenging material for students to understand is Logarithmic material. In addition, Kusuma & Masduki (2016) stated that 81% of students scored below the KKM, especially on logarithmic material. Marjan & Nur's research (2020) discusses students' difficulties in applying logarithmic properties and solving logarithmic subject mathematics problems in class X SMA. This study's results indicate a conceptual error in the logarithmic material, which has the highest percentage compared to other categories. Ulfa & Kartini's (2021) research shows that the percentage of conceptual errors is the 2nd highest after procedural errors.

Learning concepts and principles in mathematics is essential for understanding mathematical material. According to (Widodo., 2016; Hardiansyah, Manulang, & Purba, 2022; Tata & Haerudin, 2022), understanding concepts and principles in mathematics will affect students' problem-solving. Conceptual errors are often found because students only like to learn formulas and problem-solving rather than studying the concepts and forms of logarithms themselves (Pawestri, Soeyono, & Kurniawati, 2013).

Logarithmic calculation operations cause various problems among students in the learning process. The main problem for students in solving logarithmic problems is the need for more understanding of various types of logarithmic properties but only focusing on memorizing them (Donuata & Pratama, 2021). In addition, based on previous research, Pujilestari (2018) and Kusuma & Masduki (2016) discussed the same research results, namely an analysis of student errors in solving logarithmic problems based on conceptual, procedural, and technical errors. However, it does not represent the actual conditions experienced by students because it does not display excerpts of student worksheets for solving logarithmic problems. Therefore, this study will display snippets of student work and represent the location of student errors in solving logarithmic problems.

Logarithmic problems involve the use of the concept of properties and require students to understand them as a prerequisite in solving logarithmic problems. The main problem in solving problems related to mathematical concepts is significant for students. Kastolan in Sulistyaningisih & Rakhmawati (2017) explains that there are 3 types of mathematical errors, namely conceptual errors, procedural errors, and technical errors. So the probability of errors that often occur in logarithmic problems can be reviewed using Kastolan error analysis. Based on student errors in solving logarithmic problems, the researcher wants to examine further the problems experienced by students in logarithmic material. Therefore, this study aimed to analyze student errors in the logarithmic material based on its properties and the analysis of Kastolan errors.

II. Research Method

This research is a type of qualitative research. The research was conducted to identify
and uncover errors made by students in solving logarithmic problems using the Kastolan stages.

The research location is in SMA Negeri 15 Pekanbaru City, Riau Province. This school uses the 2013 curriculum in its learning process. Basic logarithmic operations are taught in class X.

The subject of this research is class X MIPA 2, totaling 18 people. The sampling technique in this study was purposive sampling. The approach in this study uses kastolan error analysis. The use of the approach to identify the types of student errors is done by looking at the completion steps. So indicators are needed to make identifying the kinds of student errors easier. The indicators used in the Kastolan error analysis approach were adopted from (Ulfa & Kartini, 2021). The indicators are presented in Table 1 as follows.

Table 1. Fault types and indicators based on the castoran error analysis procedure

<table>
<thead>
<tr>
<th>error type</th>
<th>error indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual Error</td>
<td>1. Unable to interpret questions/use a term, concept, and principle</td>
</tr>
<tr>
<td></td>
<td>2. Unable to choose the formula/properties of logarithms correctly</td>
</tr>
<tr>
<td></td>
<td>3. Unable to apply and generalize formulas/logarithmic properties correctly</td>
</tr>
<tr>
<td>Procedural Error</td>
<td>1. Mismatch of steps in solving the ordered questions</td>
</tr>
<tr>
<td></td>
<td>2. Cannot solve the problem in the simplest form</td>
</tr>
<tr>
<td>Technical Error</td>
<td>1. Error in arithmetic operations</td>
</tr>
<tr>
<td></td>
<td>2. Errors in moving numbers or arithmetic operations from one step to the next</td>
</tr>
</tbody>
</table>

In this study, the main instrument was the researcher and the basic logarithm operation test questions. The researcher examined the types of errors made by students by describing the parts of the errors that occurred. Data collection is based on the analysis of student answer sheets. The method used in analyzing the data in this research is the process, according to Creswell, including (1) preparing and compiling data for analysis; (2) numbering or marking to construct descriptions and themes; (3) representation and report of findings; (4) interpretation of findings; and (5) verification of the accuracy of the findings (Dwinata & Ramadhona, 2018).

The researcher analyzed the student's answer sheets by referring to the application of logarithmic properties. Data analysis was carried out by identifying student errors using the Kastolan stages. First, the researcher prepares and organizes student answer sheets for analysis. Next, the researcher reviewed the types of student errors by providing a code to the data and describing each error. Furthermore, the researchers carried out a process of representation and interpretation of the findings of the types of student errors. They verified the accuracy of the findings by comparing the test and interview data to obtain valid information.

III. Results and Discussion

The results of the error analysis with the Kastolan stages on the primary logarithmic operation material are presented in Table 2. below.

Table 2. Recapitulation of types of errors made by students

<table>
<thead>
<tr>
<th>Error Type</th>
<th>The Frequency of Students Who Make Mistakes in Each Question</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Conceptual</td>
<td>18</td>
<td>2</td>
</tr>
<tr>
<td>Procedural</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Technical</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

1. Conceptual Error

The following is a conceptual error reviewed based on each item.

Table 3. The average percentage of conceptual error for each question

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Conceptual Error Frequency</th>
<th>Average Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18</td>
<td>72%</td>
</tr>
</tbody>
</table>
The results show that students make conceptual errors most often, which is 74.14%. In this research, questions number 1 a or b are more common, namely 18 errors with a percentage of 72%, while number 4 has 13 errors and has an average of 87%. The majority of errors made by students lead to indicators of inability to apply logarithmic properties correctly. The following are 2 errors of students who have yet to be able to correctly understand the nature of logarithms, which can be seen in the image below.

![Figure 1. Example of conceptual error (Sw-7) question number 1b](image1)

![Figure 2. Example of conceptual error (Sw-10) question number 4](image2)

Table 2 and Table 3. The results show that students make conceptual errors most often, which is 74.14%. In this research, questions number 1 a or b are more common, namely 18 errors with a percentage of 72%, while number 4 has 13 errors and has an average of 87%. The majority of errors made by students lead to indicators of inability to apply logarithmic properties correctly. The following are 2 errors of students who have yet to be able to correctly understand the nature of logarithms, which can be seen in the image below.

Based on Table 2 and Table 4, procedural error is the second most common error after conceptual error, with a percentage of 15.52%. In contrast, if viewed based on the percentage of each question, number 2 has the highest percentage, with 57%. One of the procedural errors made by students is in number 2b, which is an error that refers to an indicator of a mismatch.
of steps in solving the ordered questions. It needs to be more systematic in its solution. The mistakes made by students can be seen in the image below.

Figure 3. Example of student procedural error (Sw-01) question number 2b

Figure 3. Shows the completion results of students (Sw-01) who operate each tribal starting from $\log 0.02; \log \frac{15}{10}$; and $-\log \sqrt{0.04}$. As seen on the answer sheet, students divide into three parts to count each tribe. The problem requires solving logarithmic operations with the property $\log a + \log b = \log (ab)$ and $\log a - \log b = \log \left(\frac{a}{b}\right)$. As a result, from the point of view of the settlement procedure carried out, it is considered inappropriate. In line with the results of research by (Marjan & Nur., 2020; Marwanti, Setiawan, & Fitriani, 2022; Angraini & Prahmana, 2018; An Nu`rma & Rahaju, 2021) that conceptual errors can lead to other errors, namely procedural. This error also occurs because students need help understanding the properties of logarithms.

3. Technical Error

The following is a technical error reviewed based on each item.

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Frekuensi Kesalahan Teknik</th>
<th>Rata-rata Persentase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>14%</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>9%</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>13%</td>
</tr>
</tbody>
</table>

Table 5.
The average percentage of technical error for each question.

Based on Table 2. Overall technical errors are errors that students in this study rarely make, so the percentage obtained is 10.34%. While in Table 5, numbers 1a, 1b, and 4 have the same number of errors, namely 2, with a percentage of 8% and 13%. The mistakes that students make are making mistakes in operating calculations. The mistakes made by students can be seen in the image below.

Figure 4. Example of a student question number 1a technical error (Sw-15)

Figure 5. Examples of technical errors in question number 4 Students (Sw-4)

Figure 4 shows a technical error made by a student (Sw-15) who did the wrong arithmetic operation on $3 \log_{25} \frac{25}{25}$ and the result written is 1, so the final result is $3 \log_{1} = 0$. The error was caused by students’ misunderstanding of fractional arithmetic operations. Previous research explained that an error in arithmetic operations, namely the use of operating signs, affected the final result (Amalia & Khabibah, 2021).

In Figure 5, student (Sw-4) made an addition and subtraction operation error. The mistake is that the student adds $0.6020 + 0.3010 + 1.07918$, so the results obtained by students are 1,98218. Students need to be more careful in operating numbers, so in this case,
overall, students only focus on adding without realizing that there are signs of subtraction operations that must be carried out. In line with the results of (Pujilestari, 2018; Maulid, S, & Sahidin, 2017; Marjan & Nur, 2020) research that errors in student calculations did not pay attention to the sign of the operation used.

IV. Conclusion

The study's results and discussion of the types of student errors using Kastolan error analysis on logarithmic material explained that the first type of error was conceptual, with a percentage of 74.14%. The second error is a procedural error, with a large percentage of 15.52%, and the third type of error, technical error, gets 10.34%.

The results of the analysis of the three types of errors in the results and discussion explain that the errors often made by students are conceptual. This shows that understanding the basic properties of logarithms still needs to improve, making it difficult to solve the problems given the following type of procedural error. This type of error shows the ability of students to determine the steps and concepts that are appropriate and systematically not following the procedures that should be carried out. Moreover, minor type of error rarely made by students, namely technical errors, shows the low accuracy of students in paying attention to the operation signs used in solving problems.

Reference


Hardiansyah, C., Manulang, B., & Purba, S. C.


